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MESON PRODUCTION AT HIGH ENERGIES AND THE PROPAGATION OF COSMIC RAYS THROUGH THE ATMOSPHERE

BY

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Synopsis

An analysis of the intensity, charge composition and energy spectra of secondary cosmic ray components suggests that the average incident extraterrestrial nucleon, in its passage through the atmosphere, is repeatedly excited to one of the low lying levels of the pion-nucleon system and decays between successive excitations by the emission of several mesons. The particle distribution in the atmosphere in the energy range between a few GeV to ~ 1000 GeV can be understood and calculated accurately in terms of such decay products and their progeny; other processes of meson production play only a minor role.

The experimental data available on secondary cosmic radiation determine approximately the properties of the excited "average" baryon state, rather similar to those known to exist at accelerator energies

Probability of excitation	$s = 0.7 \pm 0.07$
Isobar mass	$M_B \geqslant 2300$ MeV
Average number of pions emitted per decay	$n_B = 3.5 \pm 0.5$
Average charge excess among decay pions	$\lvert \pi^+ - \pi^- \rvert = 0.35 \pm 0.15$
Ratio of hyperons to nucleons among the decay products	$Y/N = 7 \pm 7$ %

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I. Introduction

In this paper an attempt is made to describe the propagation of cosmic radiation in terms of a phenomenological model for high energy nuclear interactions, which is in harmony with ideas on the process of particle creation as evolved in accelerator laboratories and at the same time simple enough for accurate calculations of intensity, charge, and energy distribution of secondary particles in the atmosphere.

A new investigation of this problem seemed to be required in view of the fact that, in recent years, a number of structural features corresponding to definite quantum states have been observed in the pion-nucleon system, which are known to play an important role at low energies and may do so also at high energies. Upto 3 GeV, single and double pion production in nucleon-nucleon collisions can be understood in terms of the excitation and subsequent deexcitation of such pion-nucleon isobaric states.(1) This mode of description, which has been developed largely by Lindenbaum and Sternheimer(2), is known as the isobar model of meson production. Recently, Damgaard and Hansen(3) have presented evidence that the same process may account for the majority of particles created by 22 GeV protons.

At very high energies ($\geqslant$ 100 GeV), on the other hand, a study of the energy and angular distribution of the great majority of particles requires a different mechanism for particle creation; it suggests a kind of fireball model(28), e. g., it can be described in terms of nearly isotropic emission of low energy particles from a cloud which is approximately at rest in the centre of mass system of the colliding nucleons (see Appendix A). Cosmic ray evidence indicates that the colliding nucleons themselves do not form part of this fireball; their energy is high in the C-systems, even after collision. This can be deduced from the propagation of nucleons through the atmosphere; in the majority of encounters, a nucleon emerges which retains a large fraction of the original energy.

However, it appears that nucleons are not the only particles which move with high velocity in the centre-of-mass system after collision; there is strong evidence that, at least in a considerable fraction of collisions, a small number of pions is generated with energies high in the rest system of the fireball and low in the rest system of one of the baryons.[(4)]. It seems natural to expect strong final-state interaction between these mesons and the nucleon so that one may describe their creation as being the result of the deexcitation of a baryon isobar. The term baryon isobar will be used only in this sense. One can then divide the particles created in high energy nucleon-nucleon encounters into two phenomenologically distinct groups:

the fireball (which contains the majority of particles, all relatively slow in the centre-of-mass system), and

two other sets, each containing a nucleon as well as a small number of mesons, which are slow in the baryon system and therefore dynamically related to the nucleon.

As the primary energy increases the multiplicity, and hence the size of the fireball, is found to increase. On the other hand, the number of mesons from deexcitation of isobaric states appears to remain constant, suggesting an essentially energy independent mass distribution of the excited baryon states.

This general picture, which appears to be in accord with the energy distribution among particles emerging from high energy collisions[(3, 4, 5)], receives strong support from other cosmic ray data; as shown below, the positive to negative ratio among sea level muons and among kaons observed in balloon exposed emulsion stacks provides evidence in favour of the frequent excitation of baryons in high energy collisions.

a) The Ratio of Positive to Negative Muons.

The ratio μ^+/μ^- measured on the surface of the earth is nearly five to four at low energies and remains constant (or possibly increases) for muon energies above 100 GeV (see Fig. 8). Since the available target nuclei in the atmosphere contain protons and neutrons in equal numbers, charge symmetry requires that the observed positive excess among muons be due to the excess of protons over neutrons in the primary cosmic radiation, corresponding to an average charge excess of 0.37* positive charges per collision. The relations $\frac{\mu^+}{\mu^-} \approx \frac{5}{4}$ and $\mu^+ - \mu^- \leqslant 0.37$ show that the muons measured on the ground are descendants of a subgroup (consisting of about 3 mesons) whose members satisfy each of two conditions:

* At a given energy per nucleon, the extra-galactic nucleons consist of 87 % protons and 13 % neutrons[(6)], corresponding to an average charge excess of 0.37 per primary interaction.

they have a preferential share in the charge excess of the incident nucleon, and

they receive an abnormally high energy so as to remain distinct among the bulk of particles created in the same interaction.

Since the observed ratio μ^+/μ^- remains essentially constant over a considerable energy range, the character of the subgroup should not depend on primary energy.

The muons in this subgroup come from parent particles which receive an energy proportional to that of the incident nucleon; this can be deduced from the fact that their energy spectrum (when corrected for their decay and for the interaction of parent particles) follows the same power law as the primary and secondary nucleon components (Figs. 2 and 5).

All these conditions are satisfied automatically if one assumes that the incident baryon, emerging from a nuclear collision in the atmosphere, finds itself some of the time in an excited state from which it returns to the nucleon ground state by meson emission.

b) The Ratio of Positive to Negative Kaons.

Cosmic ray produced kaons observed in emulsion stacks furnish quite independently another indication of the existence of baryon excitation in high energy collisions. Stopping kaons show an uncommonly high positive to negative ratio $K^+/K^- \approx 20$ (7). The observations correspond, in the mirror system of nucleon-nucleon collisions, to kaons which receive more than $25\,^0/_0$ of the energy of the incident primary. Such a high fractional energy is normal if a kaon arises from the deexcitation of the forward isobar. The large positive excess follows directly from the assumption that most of the excited isobars have strangeness number zero:

Positive kaons should then be produced in processes of the type $N^* \to K + Y$, $(M > 1610\ \text{MeV})$†; kaons of negative strangeness should be much rarer because in non-strange baryon decay they can occur only as members of kaon-antikaon pairs and have much narrower production channels:

$$N^* \to (K\overline{K}) + N \qquad (M > 1960\ \text{MeV}) \text{ or}$$
$$N^* \to K + Y^{**} \to K + \overline{K} + N \qquad (M > 2010\ \text{MeV}).$$

In view of all these considerations, it seems useful to adopt a dual picture of particle generation in the high energy range relevant to most cosmic ray

† There is now strong evidence that this process contributes appreciably to positive kaon production at accelerator energies.(3)

phenomena and investigate its consequences. We consider a very simple model which incorporates the two distinct processes of meson production. The fact that it represents existing data adequately seems to point to an underlying simplicity in the high energy collision process itself. (The model is more fully discussed in Appendix A, where it will also be shown that it is compatible with all well established experimental data on high energy interactions.) The main features of the meson production processes may be summarized as follows:

1) A fairly isotropic emission of mesons from a fireball, moving with small, i. e. non-relativistic velocity in the C-system of a nucleon-nucleon collision. The number of these "pionization" mesons increases with energy, but their C-energy does not (in conformity with the well-known energy independence of transverse momenta); as a result, their energy in the L-systems is *proportional to the square root of the primary energy.*
2) Emission of mesons from various excited baryon states whose nature is independent of collision energy above $\sim$10–15 GeV. In the L-system the energy of decay mesons associated with the forward moving baryon is therefore *proportional to primary energy* (that of the backward moving baryon is non-relativistic and essentially independent of primary energy).

The relative importance of the two processes depends on the phenomenon to be studied. In this paper we confine the investigation to the intensity and energy distribution of secondary particles in the atmosphere, i. e. to the combined effect of collisions produced by primaries whose energy distribution follows the well-known and rather steep power law. In this particular case a simplification arises for purely kinematical reasons, namely:

In the case of nucleon-nucleon collisions the two processes of particle generation cannot both contribute comparable numbers to the flux of the secondary particles in the cosmic radiation; this is a consequence of the fact that the spectra of mesons from the two processes have very different dependence on primary energy and that the steepness of the primary cosmic ray spectrum emphasizes those processes in which a large fraction of the primary energy is transferred to individual secondary particles. The calculation of the relative importance in the cosmic radiation of mesons generated by nucleons in pionization and mesons generated in the deexcitation of baryon states is straightforward on the basis of this model; it is carried out in Appendix A and the result is shown in Fig. 1. One sees that, for

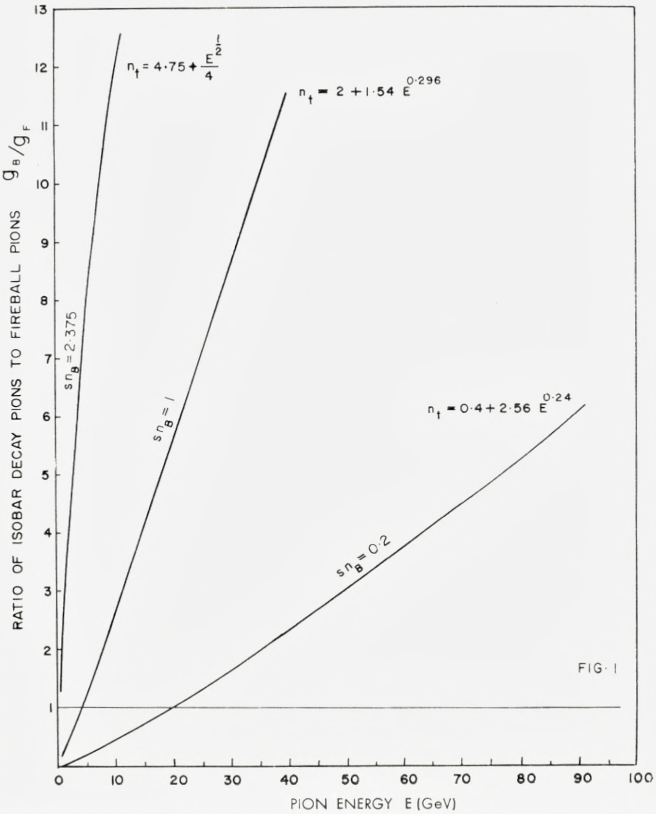

Fig. 1. Pion production from the decay of baryon isobars is compared with pion production from evaporation of a fireball (pionization). The ratio g_B/g_F (eqs. A7, A6) has been plotted for various values of the number of isobar decay pions, sn_B, which are produced in an average collision with target nuclei of low atomic weight. The total number of created particles per collision, n_t, is assumed to be related to that of charged particles, n_c, by $n_t = 1.6\,(n_c - 1) = 2\,sn_B + n_0\,E^{\varrho}$. The constants n_0 and ϱ have been chosen such that

$$n_t = \begin{cases} 6 \text{ for } E = 25 \text{ GeV} \\ 18 \text{ for } E = 2700 \text{ GeV}^{(24)} . \end{cases}$$

purely kinematical reasons, the mesons from the deexcitation of baryon isobars account for almost all particles above a few GeV even if the probability of excitation is fairly small.

We have, therefore, a situation which at first sight seems paradoxical: inspite of the fact that pions from the pionization process are more numerous than decay mesons in individual high energy collisions, *the secondary cosmic rays* observed in the atmosphere, at the surface of the earth or below ground, *represent a reasonably pure sample of the decay products of nucleon isobaric states and their progeny.*

In view of this general consequence of the assumption of isobar excitation at high energy, namely the preponderance of its deexcitation products in the atmosphere, we calculate in Section II the flux of different components of secondary cosmic radiation arising from the production and decay of isobars and neglect in the case of nucleon collisions the contribution from the pionization process, except in so far as it represents a source of energy loss for the nucleon component in the atmosphere.

The creation of particles by the collision of pions in air is included. Since the nature of these collisions is not well known, a parameter is introduced which describes essentially the degree of elasticity in pion-nucleon interactions. Except at large depths in the atmosphere, the generation of pions by pions plays only a minor role in the secondary cosmic radiation.

The charge composition p/n, $\pi^{\pm}/p$ and the ratio μ^{+}/μ^{-} as a function of the properties of the excited isobaric states are discussed in Section III.

In both Sections II and III the calculations have been carried out for a single hypothetical isobar of "average" properties. The generalization to an arbitrary set of isobars decaying in an arbitrary manner is carried out in Appendix B. With the appropriate interpretation of symbols, the formulae derived in Sections II and III remain valid for the general case.

In Section IV it is shown that one obtains a very close agreement between measured and calculated spectra of nucleons and muons over the entire energy range above a GeV by a straightforward application of this simple model, and that one obtains the experimental ratio of pions to nucleons in the lower atmosphere provided one assumes that the collisions of pions with air nuclei are essentially inelastic.

In Section V we discuss the conditions which the excitation and deexcitation of baryon states have to satisfy in order to reproduce measurements on the secondary component of cosmic radiation with regard to absolute intensity, energy distribution, and charge composition.

The results of this investigation are summarized in Section VI. The applicability of the model to collisions at ultra-high energies (i. e. air showers) remains to be investigated.

II. The Propagation of Cosmic Rays Through the Atmosphere on a General Isobar Model

The calculations are based on the following assumptions:

1) Incident and target nucleons can be treated as free. (Since nucleon-nucleon collisions exhibit a high degree of elasticity at high energy, a nucleon which enters the atmosphere as constituent of a heavy primary is not shielded effectively and will have the same interaction mean free path in the atmosphere as a primary proton).
2) Upon emerging from the collision the incident baryon has lost some energy and may find itself either in the nucleon ground state or in an excited state from which it returns to the ground state by a succession of two-body decays leading to a total emission of n_B pions, or else by the emission of heavier bosons which subsequently disintegrate into n_B pions.
3) A fixed fraction of the incident energy is used up in creating particles through the pionization process, but in the presence of mesons from the decay of baryon isobars, the energy of the particles created in the pionization process is too low to contribute significantly to the flux of secondary particles in the atmosphere, as shown in Appendix A and Fig. 1. (Also mesons from the decay of the baryon which is emitted backwards in the C-system do not contribute since their energy in the laboratory system is still lower than that of the mesons from the pionization process).

Thus, according to this model, a nucleon after collision and deexcitation will have retained a substantial fraction η of its original energy, where η does not depend on energy but does depend on the type of isobar created in the collision, on its mode of decay, and the angle at which the mesons are emitted.

In the text we shall treat the production of secondaries as if they were decay products of a single type of isobar which returns to the nucleon ground state by the emission of n_B pions. In Appendix B we treat the more general case of deexcitation of a mixture of baryon states, each according to its own decay scheme, characterized by its mass and a set of decay branching ratios. The resulting formulae for the flux of secondary particles are identical with those given in this section, provided the quantities enclosed in brackets $\langle\,\rangle$ are replaced by the appropriate average values derived in Appendix B.

II.1 *The Energy Spectrum of Nucleons in the Atmosphere*

Let the differential energy spectrum of primary cosmic ray nucleons be represented by

$$N(0, E)\, dE = S_0 \frac{dE}{E^{\gamma+1}} \tag{II.1}$$

over the entire energy range of interest.*

The number of nucleons of energy E which have suffered j collisions is given by

$$N(E)_j = N(0, E)\ \eta^{\gamma j}.$$

The probability that a nucleon has suffered j collisions by the time it has reached an atmospheric depth of x g/cm² is

$$e^{-x/\lambda} \left(\frac{x}{\lambda}\right)^j \frac{1}{j!}.$$

Therefore, the flux of nucleons of energy E at a depth x g/cm² is given by

$$N(x, E) = N(0, E)\, e^{-x/\lambda} \sum_{j=0}^{\infty} \left(\frac{x}{\lambda}\right)^j \frac{\eta^{\gamma j}}{j!} = N(0, \mathrm{E})\, e^{-x/\Lambda}, \tag{II.2}$$

where $\Lambda = \dfrac{\lambda}{1 - \langle \eta^\gamma \rangle}$ is the attenuation length of nucleons in air and λ their interaction mean free path.

The bracket around $\langle \eta^\gamma \rangle$ indicates, as explained before, that it will have to be replaced by an appropriate average (eq. B.9).

Various corrections to eq. II.2 are required in the low energy region; they will be discussed in Appendix D (eq. D 2, 3).

II.2 *The Production Spectrum and Flux of Charged Pions*

The production spectrum of charged pions from baryon decay can be calculated in a straightforward manner on the basis of the model (see Appendix B) and is given by

$$P_\pi(x, E)\, dx = \langle B \rangle\, N(0, \mathrm{E})\, e^{-x/\Lambda} \frac{dx}{\lambda}. \tag{II.3}$$

$\langle B \rangle$ is defined by relation eqs. B.15, 17 in terms of the relative production rates of different isobars and their decay properties.

* Effects due to the apparent steepening of the primary spectrum above $\sim 10^{14}$ eV can be observed at present only in extensive air shower frequencies and perhaps near the upper end of the γ-ray spectrum; this effect will not be considered here.

(For a single isobar with transition directly to the ground state by isotropic emission of a charged pion $\langle B \rangle = B$ and is equal to

$$B = (\eta' \varepsilon)^{\gamma} \frac{(1+\beta)^{\gamma+1} - (1-\beta)^{\gamma+1}}{2\beta(\gamma+1)}, \tag{II.4}$$

where η' is the fraction of the incident nucleon energy retained by the isobar after pionization, ε is the fractional energy in the isobar rest system which is carried away by the decay pion, β its velocity, and γ is the exponent of the primary spectrum).

The interactions of pions in the atmosphere represent another source of particle creation; this increases the pion flux significantly in the lower parts of the atmosphere. Since the nature of pion-nucleon collisions is not well known, one must introduce a parameter which describes essentially the degree of elasticity which characterizes such collisions.

Complete elasticity implies that practically the entire energy is carried away by a single pion. Complete inelasticity implies that the available energy in the C-system is shared in a non-preferential manner by all the created mesons. (Unless the C-system energy of created pions is assumed to decrease with increasing collision energy, maximal inelasticity means that pion multiplicity is proportional to the square root of the incident energy). One can describe these extreme as well as intermediate conditions by assuming that (apart from a possible excitation of the target baryon) collisions of pions with nucleons lead on the average to the creation of

$$\nu = \nu_0 E_0^{\left(\frac{t-1}{t}\right)} \tag{II.5}$$

mesons and that the incident energy in the L-system is shared by half of them, so that their energy is

$$E = \frac{2 E_0}{\nu}. \tag{II.6}$$

The production of charged pions by pions is then given by

$$P'_{\pi}(x, E) = (q_+ + q_-) \frac{t}{\lambda_{\pi}} (KE)^{2(t-1)} F_{\pi}\left[x, \frac{(KE)^t}{K}\right], \tag{II.7}$$

where

$$K = \left(\frac{\nu_0}{2}\right)^{\frac{t}{t-1}},$$

$q_{\pm 0}$ is the fraction of created pions of different charge which share the available energy, and $F_{\pi}(x, E)$ is the flux of charged pions at depth x with energy E.

This flux of charged pions is given by the solution of the differential equation

$$\frac{dF_\pi}{dx} + F_\pi\left(\frac{1}{\lambda_\pi} + \frac{u}{x}\right) = P_\pi + P'_\pi, \tag{II.8}$$

where $u = \frac{h_0 m_\pi}{c\tau_\pi E} = \frac{\varepsilon_\pi}{E}$ ($\varepsilon_\pi = 128$ GeV if $h_0 = 7$ km is the scale height of the atmosphere); λ_π is the interaction mean free path of pions in air.

The extreme cases are represented by $t = 1$ (elastic) in which case the equation reduces to

$$\frac{dF_\pi}{dx} + F_\pi\left(\frac{q_0}{\lambda_\pi} + \frac{u}{x}\right) = P_\pi, \tag{II.9}$$

and $t = 2$ (complete inelasticity).

The exact solution of equation II.8 may be written in the form

$$F_\pi(x, E) = \frac{S_0\langle B\rangle}{E^{\gamma+1}}\frac{x}{\lambda}e^{-x/\lambda_\pi}\sum_{i=0}^{\infty} a_i(E)\left(\frac{x}{\lambda_\pi}\right)^i, \tag{II.10}$$

where

$$a_i = \sum_{n=0}^{i} \frac{[(q_+ + q_-)\,t]^n}{(KE)^{(\gamma-1)\,(t^n-1)}}\,\frac{\left(1-\frac{\lambda_\pi}{\Lambda}\right)^{i-n}}{(i-n)!}\left[\prod_{j=0}^{n}\left(1+i-j+\frac{u}{(KE)^{t^j-1}}\right)\right]^{-1}. \tag{II.11}$$

This can be verified by substitution. The pions which come directly from isobar decay are represented by the term $n = 0$; higher terms are important only in the lower atmosphere and for pions of intermediate energy (10–100 GeV).

II.3 *The Production of Neutral Pions*

The production spectrum of neutral pions is obtained from eqs. II.3 and II.7:

$$P_{\pi^0}(x, E) = \frac{1}{2}P_\pi(x, E) + \frac{q_0 t}{\lambda_\pi}(KE)^{2(t-1)}F_\pi\left[x, \frac{(KE)^t}{K}\right]. \tag{II.12}$$

From this equation the production spectrum and flux of γ-rays can be calculated in a straightforward manner if one assumes that γ-rays arise primarily from the decay of neutral pions.

II.4 *The Flux of Muons*

The production spectrum of muons from pion decay is

$$P_\mu(x, E) = \frac{\varepsilon_\pi}{xE} F_\pi[x, (rE)]. \tag{II.13}$$

Here, in analogy to eq. II.4,

$$r = \frac{2}{1 + \frac{m_\mu^2}{m_\pi^2}} \left[\frac{2\beta(\sigma+1)}{(1+\beta)^{\sigma+1} - (1-\beta)^{\sigma+1}}\right]^{\frac{1}{\sigma}} \approx 1.27, \tag{II.14}$$

where $\beta = 0.28$ is the muon velocity in the pion rest system

and
$$-[\sigma(x, E) + 1] = \frac{d[\log F_\pi(x, E)]}{d \log E} \tag{II.15}$$

is the exponent for the "best fitting" power law describing the pion spectrum. (Since σ varies slowly between γ and $\gamma+1$, the expression in brackets in II.14 is very close to unity and can be neglected; this is equivalent to the assumption that each muon receives 79 % of the pion energy, irrespective of the angle of emission).

The probability that a muon, produced at atmospheric depth z with energy E_z survives until it reaches depth x while losing energy by ionization at the rate b, so that it arrives with energy $E = E_z - b(x-z)$ is given by

$$\omega(z, E_z; x) = \left[\frac{z}{x}\left(1 - \frac{b(x-z)}{E_z}\right)\right]^{\frac{\varepsilon_\mu}{E_z + bz}} \tag{II.16}$$

$$\left(\varepsilon_\mu = \frac{h_0 m_\mu}{c\tau_\mu} \approx 1.12 \text{ GeV}\right).$$

Thus the muon flux is

$$\left.\begin{aligned} F_\mu(x, E) &= \int_0^x dz\, \omega P_\mu(z, E_z) \\ &= \frac{S_0 \langle B\rangle \varepsilon_\pi}{\lambda r^{\gamma+1}} \left(\frac{\lambda_\pi E}{x}\right)^v \frac{\sum_{i=0}^{\infty} a_i(rE')}{E'^{\gamma+2+v}} \int_0^{x/\lambda_\pi} dy\, e^{-y} y^{i+v}, \end{aligned}\right\} \tag{II.17}$$

where the integral can be replaced by the gamma function $\Gamma(i+v+1)$ for $x \gg \lambda_\pi$,

$$v = \frac{\varepsilon_\mu}{E + bx},$$

$$E' = E + b(x - \bar{x})$$

and the "mean height of production"

$$\bar{x} = \frac{\int_0^x dz z \omega P_\mu(z, Ez)}{\int^x dz \omega P_\mu(z, E_z)} = \lambda_\pi (1 + v) + \frac{\lambda_\pi \sum_{i=0}^{\infty} i a_i (rE') \Gamma(i + v + 1)}{\sum_{i=0}^{\infty} a_i (rE') \Gamma(i + v + 1)} \qquad \text{(II.18)}$$

for $x \gg \lambda_\pi$.

When $\lambda_\pi \approx \Lambda$, the first term in the summation, i. e. $a_0 = \frac{1}{1 + \varepsilon_{\pi/rE'}}$, accounts for more than 80 % of the muons in the lower atmosphere at all energies.

II.5 *The Flux of Neutrinos from $\pi - \mu$ Decay*

The neutrino flux has the same form as the muon flux, but without the terms due to ionization and decay; it is, therefore, obtained from the muon flux (eq. II.17) by setting $v = 0$ and $b = 0$ and replacing r by

$$r' = \frac{2}{1 - \left(\frac{m_\mu}{m_\pi}\right)^2} \frac{(1 + \sigma)^{\frac{1}{\sigma}}}{2} \simeq 4.0 \pm 0.2 \qquad \text{(II.19)}$$

for

$$\gamma < \sigma < \gamma + 1 .$$

$$F_\nu (x, E) = \frac{S_0 \langle B \rangle \varepsilon_\pi}{(\lambda/\lambda_\pi) r'^{\gamma + 1}} \frac{\sum_{i=0}^{\infty} a_i (r' E) i!}{E^{\gamma + 2}} . \qquad \text{(II.20)}$$

III. The Charge Composition of Secondary Cosmic Rays

The target nuclei in the atmosphere contain equal numbers of protons and neutrons. If the incident cosmic ray beam also were charge symmetric, then all secondary components of cosmic radiation would have to exhibit charge symmetry on any model of particle creation. Actually, the primary cosmic ray particles bring a known amount of excess positive charge. The manner in which this excess is shared by various secondary components of the cosmic radiation provides clues to the nature of high energy interactions and, in particular, to the excitation and decay modes of isobars. In this section we investigate the charge composition of different components of cosmic radiation on a general isobar model.

III.1. *The Neutron to Proton Ratio in the Atmosphere*

Let w represent the probability that a nucleon after colliding with a charge symmetric target and after possible excitation and decay emerges in a different charge state. After j collisions the original composition of the nucleon beam† δ_0 will be changed into

$$\delta_j = \left(\frac{p-n}{p+n}\right)_j = \delta_0 (1-2\,w)^j. \qquad \text{(III.1)}$$

Proceeding as in the derivation of eq. (II.2) one finds

$$\delta_x = \frac{N_p - N_n}{N} = \delta_0 e^{-x\left(\frac{1}{\lambda}-\frac{1}{\Lambda}\right)} \sum_{j=0}^{\infty} \left(\frac{x\,\eta^\gamma}{\lambda}\right)^j \frac{(1-2\,w)^j}{j!} = \delta_0 e^{-\frac{2\,x}{\lambda}\langle \eta^\gamma w\rangle}. \qquad \text{(III.2)}$$

Thus

$$N_p(x, E) = N(0, E)\, e^{-\frac{x}{\Lambda}} \left(\frac{1+\delta_x}{2}\right), \qquad \text{(III.3)}$$

$$N_n(x, E) = N(0, E)\, e^{-\frac{x}{\Lambda}} \left(\frac{1-\delta_x}{2}\right). \qquad \text{(III.4)}$$

An explicit expression for $\langle \eta^\gamma w\rangle$ in terms of isobar properties is given in Appendix B (eq. B.10).
At energies $E \leqslant 10$ GeV eqs. (III.3, 4) have to be corrected as discussed in Appendix D.

III.2. *The Ratio of Positive to Negative Pions and Muons*

Let $\delta_\pi = \dfrac{n_+ - n_-}{n_+ + n_-}$ be the composition of the charged pions emitted in the deexcitation of a baryon which entered the collision as a proton. (Because

† The composition of the primary beam $\delta_0 = 0.74$ is known[6] to be constant within experimental error upto nucleon energies of order 10^{13} eV. At still higher energies there is evidence for a steepening of the primary spectrum and, if real, this will be accompanied presumably by changes in the chemical composition and the proton to neutron ratio.[8]

of charge symmetry, this changes sign if the incident particle is a neutron). The composition of the decay pions produced at depth x is then the product

$$\frac{P_{\pi^+} - P_{\pi^-}}{P_{\pi^+} + P_{\pi^-}} = \delta_x \langle \delta_\pi \rangle, \tag{III.5}$$

where $\langle \delta_\pi \rangle$ is the average positive excess mong the decay pions; it is defined by eqs. (B.15, 16) for an arbitrary mixture of isobaric states.

In order to determine the charge composition of the pion flux in the atmosphere, one may rewrite (eq. II.8.)

Setting

$$\left.\begin{aligned} F_\pi &= F_{\pi^+} + F_{\pi^-} \\ \text{and} \qquad G_\pi &= F_{\pi^+} - F_{\pi^-} \end{aligned}\right\} \tag{III.6}$$

one finds

$$\left.\begin{aligned} &\frac{dF_\pi(x,E)}{dx} + F_\pi(x,E)\left(\frac{1}{\lambda_\pi} + \frac{u}{x}\right) \\ &\qquad = \frac{S_0 \langle B \rangle e^{-x/\Lambda}}{\lambda E^{\gamma+1}} + (q_+ + q_-)\frac{t}{\lambda_\pi}(KE)^{2(t-1)} F_\pi\left[x, \frac{(KE)^t}{K}\right] \end{aligned}\right\} \tag{III.7}$$

and

$$\left.\begin{aligned} &\frac{dG_\pi(x,E)}{dx} + G_\pi(x,E)\left(\frac{1}{\lambda_\pi} + \frac{u}{x}\right) \\ &\qquad = \delta_x \langle \delta_\pi \rangle \frac{S_0 \langle B \rangle}{\lambda E^{\gamma+1}} e^{-x/\Lambda} + (q_+ - q_-)\frac{t}{\lambda_\pi}(KE)^{2(t-1)} G_\pi\left[x, \frac{(KE)^t}{K}\right]. \end{aligned}\right\} \tag{III.8}$$

q_+, q_- is the fraction of positives or negatives among the pions which share the energy available in a pion induced interaction for the case that the incident pion had *positive* charge. Charge symmetry requires that these quantities change sign if the incident pion is negative; on the other hand, charge conservation in collisions of pions with charge symmetric targets requires that

$$\frac{1}{2} \leqslant (q_+ - q_-)\frac{\nu}{2} = (q_+ - q_-)\left(\frac{KE}{2}\right)^{t-1} \leqslant 1, \tag{III.9}$$

depending on whether the excess charge brought in by the incident pion is uniformly distributed among all secondaries or appears preferentially among the more energetic ones. Making use of eqs. (III.2, 3, 4 and 7), eq. (III.8) can now be rewritten in a form similar to the differential equation (III.7:

$$\left.\begin{aligned} &\frac{dG_\pi(x, E)}{dx} + G_\pi(x, E)\left(\frac{1}{\lambda_\pi} + \frac{u}{x}\right) \\ &= \delta_0 \langle \delta_\pi \rangle \frac{S_0 \langle B \rangle}{\lambda E^{\gamma+1}} e^{-x/\Lambda'} + \frac{\alpha t}{4\lambda_\pi} (KE)^{t-1} G_\pi\left[x, \frac{(KE)^t}{K}\right], \end{aligned}\right\} \text{(III.8 a)}$$

where

$$\frac{1}{\Lambda'} = \frac{1}{\Lambda} + 2\frac{\langle \eta^\gamma w \rangle}{\lambda} \quad \text{(III.10)}$$

and

$$1 \leqslant \alpha \leqslant 2.$$

In analogy to eq. (II.10) the solution can be written in the form

$$G_\pi(x, E) = \frac{\delta_0 \langle \delta_\pi \rangle S_0 \langle B \rangle}{E^{\gamma+1}} \frac{x}{\lambda} e^{-x/\lambda_\pi} \sum_{i=0}^{\infty} a_i'(E) \left(\frac{x}{\lambda_\pi}\right)^i, \quad \text{(III.11)}$$

where a_i' can be obtained from a_i by making the substitutions

$$\Lambda \to \Lambda'$$

$$\gamma \to \gamma + 1$$

$$(q_+ + q_-)\, t \to \frac{\alpha}{4}.$$

Thus, the charge composition of the pion flux is

$$\frac{F_{\pi^+} - F_{\pi^-}}{F_{\pi^+} + F_{\pi^-}} = \delta_0 \langle \delta_\pi \rangle \frac{\sum_{i=0}^{\infty} a_i'(E) \left(\frac{x}{\lambda_\pi}\right)^i}{\sum_{i=0}^{\infty} a_i(E) \left(\frac{x}{\lambda_\pi}\right)^i} \quad \text{(III.12)}$$

and that of the muons from pion decay (see II.17)

$$\delta_\mu = \frac{F_{\mu^+} - F_{\mu^-}}{F_{\mu^+} + F_{\mu^-}} = \delta_0 \langle \delta_\pi \rangle \frac{\sum_{i=0}^{\infty} a_i'(rE') \int_0^{x/\lambda_\pi} dy\, e^{-y} y^{i+v}}{\sum_{i=0}^{\infty} a_i(rE') \int_0^{x/\lambda_\pi} dy\, e^{-y} y^{i+v}}. \quad \text{(III.13)}$$

The Effect of Kaons on the Muon Charge Ratio at Sea Level.

In this and the preceding section it was tacitly assumed that pions are the only particles which, by their decay, contribute to the muon flux. However, non-strange isobars of sufficiently large mass can also decay into a kaon and a hyperon, or a kaon-antikaon pair and a non-strange baryon; by subsequent decay these particles contribute to the observed flux of pions, γ-rays, and muons.

In general, one expects that such decay modes make only a moderate contribution to the flux of secondaries in the atmosphere. In the case of high energy muons, however, the part of the contribution which does not involve an intermediate pion (viz. the process $K \rightarrow \mu + \nu$) is amplified because the mean life of charged kaons is significantly shorter than that of charged pions and their mass is greater. Therefore, at a given energy, the probability of decay before interacting in the atmosphere is larger for a kaon than for a pion and, in the energy range where pion interaction becomes more probable than decay, an increasing fraction of muons arises from the decay of charged kaons.

Also hyperons and neutral K-particles can decay into muons without an intermediate pion, but they will contribute little, because in the case of hyperons and K_1^0 the branching ratio is small, and in the case of K_2^0 the lifetime is long. Therefore, it is the presence of charged kaons decaying directly into muons which produces the largest effect.

Among the decay modes which contribute kaons, the mode

$$N^* \rightarrow K + Y$$

is no doubt dominant;† it can produce positive but not negative kaons.

(Negative kaons can arise in two ways; either through $N^* \rightarrow (K\overline{K}) + N$ or through $N^* \rightarrow K + Y^*$, where the hyperon state is highly excited and therefore has a certain probability for fast decay via $Y^* \rightarrow \overline{K} + N$).

The ratio of positive to negative muons becomes therefore

$$\frac{\mu^+}{\mu^-} = \frac{1+\delta_\mu}{1-\delta_\mu} + \frac{2}{1-\delta_\mu} b_K \frac{F_{\mu_K}}{F_\mu}, \qquad \text{(III.14)}$$

† As pointed out in Section I, the strong excess of positive over negative kaons among the very slow and the very fast particles produced in nuclear interactions indicates that modes which produce only kaons of strangeness +1 dominate in the decay of non-strange isobars.

where b_K is the branching ratio for the decay mode $N^* \to K + Y$ and F_μ, δ_μ, and F_{μ_K} are given by eqs. (II.17, III.13 and C.6), respectively. $\frac{F_{\mu_K}}{F_\mu}$ depends on energy approximately as

$$\frac{1+\frac{E}{\varepsilon_\pi/r}}{1+\frac{E}{\varepsilon_K/r_K}} = \left(\frac{1+\frac{E}{100}}{1+\frac{E}{570}}\right)_{E \text{ in GeV.}} \qquad \text{(III.15)}$$

IV. Comparison between Measured and Calculated Properties of the Secondary Cosmic Radiation

In the previous section explicit expressions have been obtained for the altitude and energy dependence of protons, neutrons, pions, muons, and neutrinos.

It remains to assign numerical values to the various *energy independent* parameters which characterize the primary cosmic radiation and the interactions of nucleons and pions in the atmosphere. In principle, it should be possible to get the information on interaction cross sections from the asymptotic behaviour of these particles in the upper energy range of present-day accelerators. However, some of these data, especially on the formation of isobars, are not yet known adequately and it is necessary to use a few cosmic ray measurements to assign numerical values to some of these constants. The following values have been used for drawing Figs. 2–6.

a) The interaction mean free path of nucleons in air

$$\lambda = 75 \pm 5 \text{ g/cm}^2 \dagger$$

has been obtained from the absorption in graphite of neutrons capable of producing charged penetrating particles [(10)].

b) The interaction mean free path for pions in air

$$\lambda_\pi = 120 \text{ g/cm}^2$$

† Bad geometry absorption measurements at accelerator energies[9]) yield a mean free path which is at least 20 % longer. It is difficult to account for this discrepancy, unless it is due to interactions in which the incident proton suffers only a small energy loss and receives a transverse momentum less than 150 MeV/c. Such collisions could be missed in the accelerator experiment because the emerging proton falls within the diffraction peak.

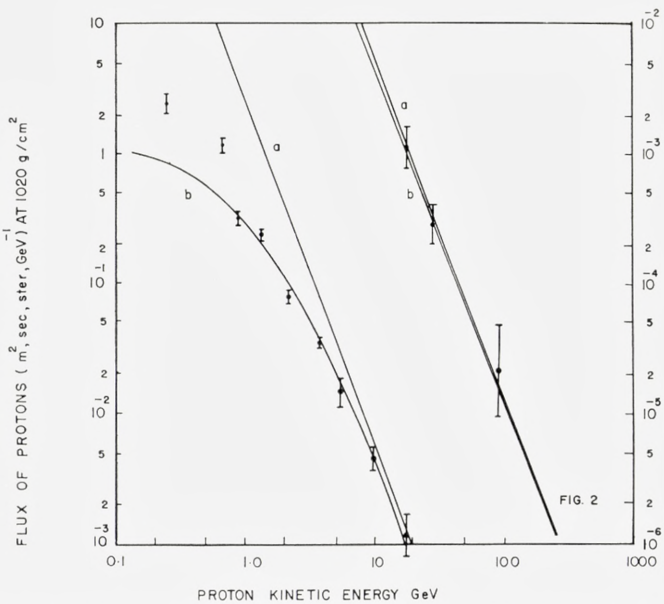

Fig. 2. The energy spectrum of protons at sea level. Curve 'a' is a line of constant slope $\gamma + 1 = 2.67$. Curve 'b' represents the calculated spectrum (eq. D.3) based on an attenuation mean free path $\Lambda = 120$ g/cm^2 and an ionization loss of 2 MeV (g/cm^2). The experimental points are taken from ref. 12. The excess of observed protons at low energy is of the right order of magnitude to be attributed to terrestrial protons from the target nuclei in the atmosphere (see Appendix Db).

has been estimated by multiplying λ with the ratio of the cross section $\frac{\sigma_{pp}}{\sigma_{\pi p}} \approx 1.6$ obtained in high energy laboratories (11).

c) The exponent of the primary spectrum

$$\gamma = 1.67$$

is consistent with direct measurements at the top of the atmosphere; the exact value has been chosen so as to give the best fit to the sea level proton spectrum at high energy (12).

d) The absorption length for nucleons in the atmosphere

$$\Lambda = 120 \pm 5 \text{ g/cm}^2$$

has been obtained from the absolute intensity of protons at sea level as measured by the Durham group (12), and the absolute value of the primary cosmic ray flux as given by McDonald and Webber (13). The expected deviations of the proton spectrum from a simple power law at low energies are discussed in Appendix D.

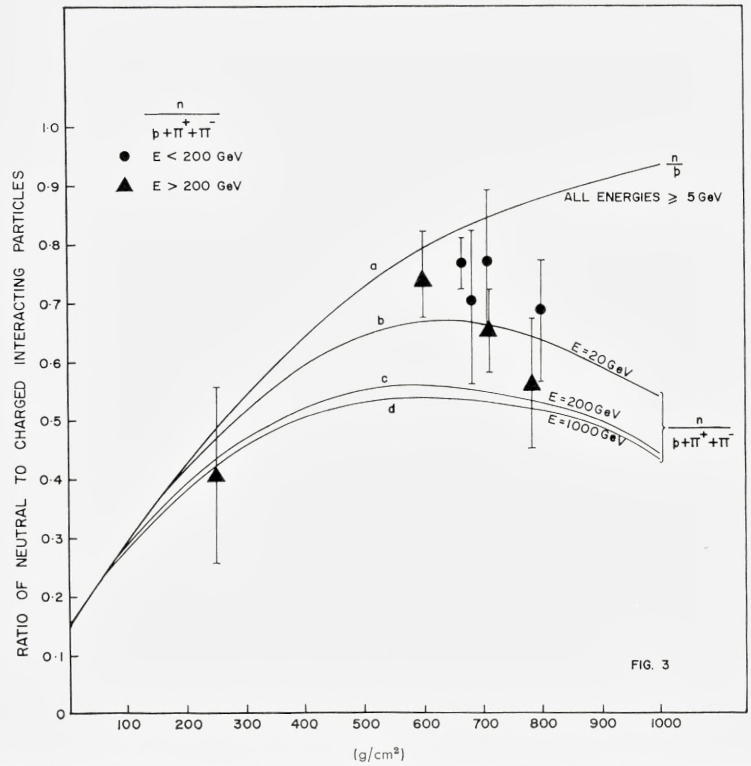

Fig. 3. Curve 'a' represents, as a function of altitude, the ratio of neutrons to protons at all energies for which ionization loss and contribution of recoil nucleons (Appendix D) can be neglected. Curves 'b', 'c', and 'd' represent the ratio of neutrons to the sum of protons and charged pions for various energies. The experimental data are those of ref. 26.

e) The composition of the primary radiation

$$\delta_0 = \frac{p_0 - n_0}{p_0 + n_0} = 0.74 \pm 0.01$$

has been obtained from the measurements of the primary chemical composition as reviewed by WADDINGTON (6).

f) The charge exchange probability for nucleons colliding in the atmosphere

$$w = 0.3 \begin{pmatrix} +0.2 \\ -0.08 \end{pmatrix}$$

has been chosen to reproduce the neutron to proton ratio as measured at mountain altitude (14). (It is assumed that the average $\langle \eta^{\gamma} w \rangle$ can be replaced by $\langle \eta^{\gamma} \rangle w$).

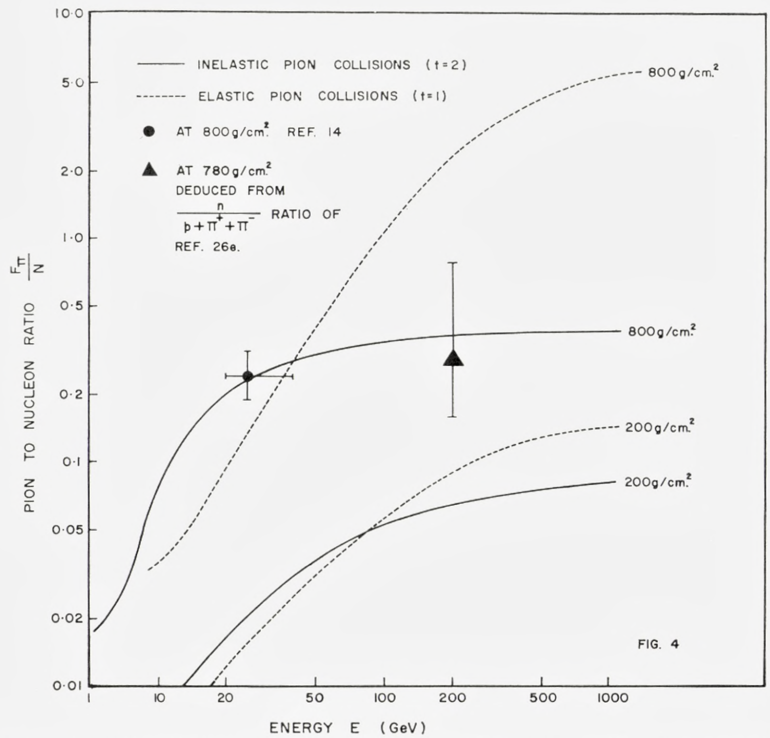

Fig. 4. The ratio of charged pions to nucleons as a function of energy at atmospheric depths of 200 g/cm² and 800 g/cm². The solid curves are calculated on the assumption of complete inelasticity in pion-nucleon collisions, the dotted curves for complete elasticity.

g) The constants which determine the number of pions which share the energy in pion nucleon collisions, eq. (II.5), have been chosen to represent the extreme cases

α) Complete elasticity $t = 1$ (for $t = 1$ the equations are independent of K)

β) Complete inelasticity $t = 2$ $\qquad \nu = 0.7\, E^{\frac{1}{2}}_{\mathrm{GeV}}$ (IV.1)

$$\left[\text{i. e. } K = \left(\frac{\nu_0}{2}\right)^2 = \frac{1}{8}\right].$$

K is chosen to fit the measured $\pi^{\pm}/p$ ratio at 800 g/cm² in the energy region 20–40 GeV [14].

h) The constant $\langle B \rangle$ which characterizes the fractional energy given to pions in the decay of baryon isobars is obtained by comparing calculated and measured sea level flux of muons at 40 GeV [15]

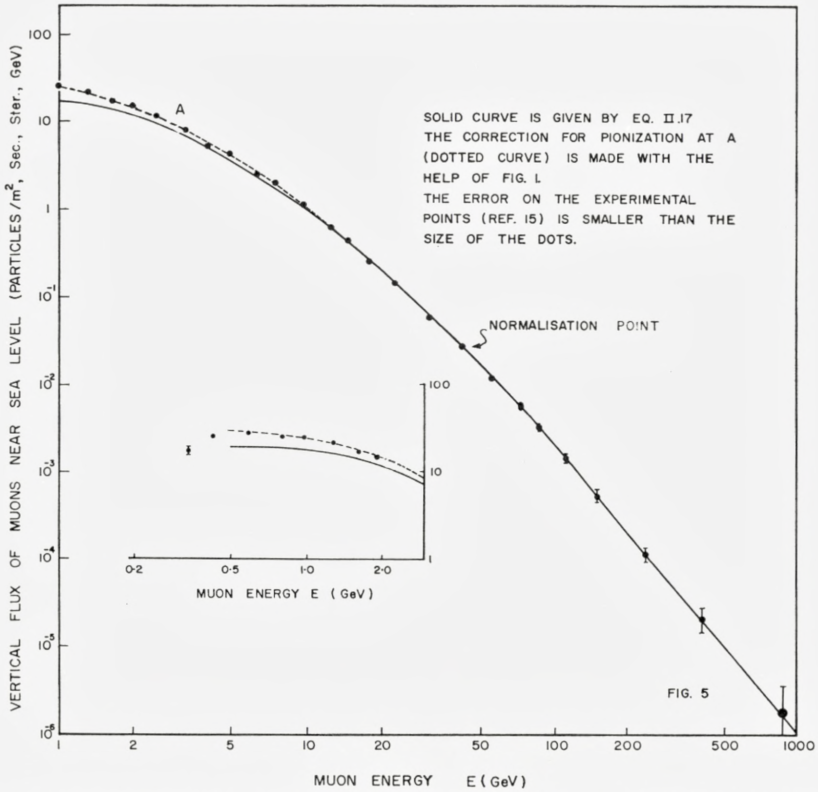

Fig. 5. The energy spectrum of muons at sea level. The solid curve represents the calculated spectrum, eq. II.17, normalized at $E \approx 40$ GeV. The dotted curve represents the spectrum after a rough correction for contribution from pionization has been made with the help of Fig. 1. the closeness of its fit to the experimental data is therefore somewhat fortuitous.

$\langle B \rangle = (3.35 \pm 0.3) \cdot 10^{-2}$ (both for the elastic case and for the inelastic case).

Using these constants the following curves have been calculated and reproduced together with the available experimental points:

α) The energy spectrum of protons Fig. 2 (eq. D3 (Appendix D)).

β) The ratio of neutrons to protons as a function of atmospheric pressure Fig. 3 a (eqs. III.3, 4)

γ) The ratio of neutral to charged interacting particles $\dfrac{n}{p+\pi^{+}+\pi^{-}}$ as

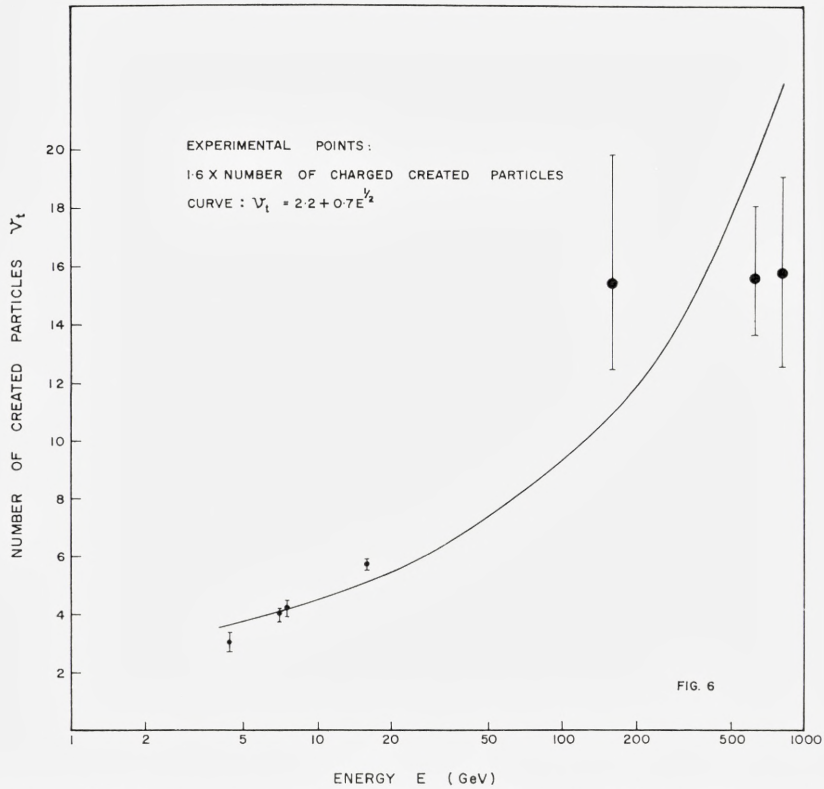

Fig. 6. The total number of created particles, charged and neutral, emitted in collisions of charged pions with target nuclei of low atomic weight, is plotted as a function of energy. The curve represents the relation IV.1 a, with $sn_B \approx 2.4$. Experimental points are from the data compiled in ref. 27.

a function of atmospheric pressure for various energies, on the assumption that pion-nucleon collisions are completely inelastic. Figs. 3 b, c, d (eqs. II.10 and III.3, 4).

δ) The ratio of pions to nucleons at an atmospheric depth of 200 g/cm² and 800 g/cm² for the two extreme cases: that pion nucleon interactions are completely elastic and that they are completely inelastic. Fig. 4 (eqs. II.10 and III.3).

ε) The energy spectrum of muons at sea level. Fig. 5 (eq. II.17).

φ) The multiplicity of created particle in pion nucleon collisions. Fig. 6.

$$\nu_t = sn_B + 0.7\, E^{\frac{1}{2}}_{(\mathrm{Gev})}. \qquad (\mathrm{IV.1a})^{\dagger}$$

† The first term has been added to eq. (IV.1) to represent low energy mesons from the decay of an excited target baryon.

Formula (IV. 1 a), which corresponds to completely inelastic pion-nucleon interactions at high energy, cannot be expected to reproduce the multiplicity accurately at low energy; however, it describes the available data reasonably well provided one choses the mean number of mesons contributed by the target baryon, $sn_B \approx 2$ (Fig. 6). [The experimental data for high energy pion collisions are extremely poor; data below 20 GeV seem to indicate that the multiplicity of created particles increases not slower than $E^{\frac{1}{2}}$, independent of the choice of sn_B].

As shown in Fig. 4, the difference in the pion flux for the assumption of complete elasticity and complete inelasticity is small at 200 g/cm^2 and, therefore, the muon spectra at sea level are insensitive to the nature of pion-nucleon interactions. On the other hand, the pion flux near ground is very sensitive; compared to the elastic case the assumption of strong multiplication of pions leads to a large excess between 10 GeV and ~30 GeV and a very large deficit above ~50 GeV. The inelastic case is in better agreement with the existing determination of the pion-nucleon ratio than the elastic case.

Assuming then a high degree of inelasticity in pion interactions, the comparison between calculated and measured quantities (Figs. 2–6) shows that the very simple, energy independent model of high energy collisions, which has been adopted, is adequate for describing the distribution of secondary cosmic radiation within the accuracy of existing measurements.

It is now of interest to discuss the restrictions which are imposed on the masses, the excitation probabilities, and the decay modes of isobars by the numerical values of $\langle B \rangle$ and Λ and by the observed energy dependence of multiplicity of created particles in nucleon-nucleon collisions.

V. Average Parameters Characterizing the Production and Properties of the Dominant Baryon Isobars

In order to find the simplest isobar model capable of accounting for all existing observations on secondary cosmic radiation, one may assume tentatively that the incident nucleon, after having lost a fixed fraction $(1-\eta')$ of its energy in the pionization process, has a probability of emerging as an excited baryon of mass M_B and a probability $(1-s)$ of emerging in the nucleon ground state. If excited, the baryon is assumed to decay to the ground state by the isotropic emission of n_B mesons, all of which have the same energy in the baryon rest system.†

† These n_B mesons may of course be themselves decay products of a boson isobar.

V.1. *The Excitation Probability of Isobars and the Average Number of Decay Mesons*

The average number of mesons emitted in the decay of nuclear isobaric states can be estimated from the shape of the multiplicity-energy relation for particles created in nucleon-nucleon collisions.

It is shown in Appendix A.2 that the assumptions underlying the model discussed in this paper determine the form of the relation between multiplicity of created particles and energy (eq. A.4)

$$n_t = 2\, s n_B + n_0\, E^{\frac{1}{2}},$$

where the first term represents the average number of decay mesons from isobaric states of the baryon and the second the contribution from pionization. The actual relation may be expected to show some structure if the size of the heavy bosons constituting the fireball were quantized in units $M_F \gg m_\pi$.

When comparing relation A.4 with existing experimental values shown in Fig. 9, one sees that it is consistent with existing data

$$\text{for} \quad 2 < s n_B < 3$$

$$\text{and} \quad n_0 \approx \frac{1}{4}, \text{ for } E \text{ in GeV.}$$

The difference between this relation and the frequently employed empirical form $n_t \sim E^{\frac{1}{4}}$ becomes significant only at energies above $\sim 10^{15}$ eV, i. e. in the study of extensive air showers.

Separate values for the excitation probability s and the mean number of isobar decay pions n_B can be obtained by using the numerical values for $\langle \eta^\gamma \rangle$ and $\langle B \rangle$, obtained in the preceding section.

$\langle \eta^\gamma \rangle$ is related to the mean elasticity of collisions and can be expressed in terms of a ratio of nucleon interaction mean free path and attenuation length (II.2) which have been given in the previous section:

$$\langle \eta^\gamma \rangle = 1 - \frac{\lambda}{\Lambda} = 0.37 \pm 0.06 . \tag{V.1}$$

With the help of eq. (B.9) one finds (for the particular case of a single type of baryon isobar) the relation between this parameter and the isobar properties:

$$\langle \eta^\gamma \rangle = \eta'^\gamma \left[1 + s\,(A-1)\right], \tag{V.2}$$

where A (defined by B.5) is given by

$$A = \mathfrak{E}_p^\gamma \frac{(1+\beta_p)^{\gamma+1} - (1-\beta_p)^{\gamma+1}}{2\,\beta_p(\gamma+1)}.$$

$\mathfrak{E}_p = \frac{M_B^2 + M_p^2 - n_B^2 m_\pi^2}{2\,M_B^2}$ is the average fractional energy in the rest system of the isobar carried away by the deexcited nucleon and β_p is its velocity.

A second relation involving the same parameters is obtained from eqs. (B.15, 17), which determine the $\langle B \rangle$ in terms if isobar properties. ($\langle B \rangle$ is the scale factor which relates pion production to nucleon intensity).

For the case under discussion eqs. (B.15), 17 yield

$$\langle B \rangle = \tfrac{2}{3}\, s n_B\, \alpha \eta'^{\gamma} = (3.35 \pm 0.3) \times 10^{-2}; \qquad \text{(V.3)}$$

here α (defined by B.13) is given by

$$\alpha = \mathfrak{e}^\gamma \frac{(1+\beta)^{\gamma+1} - (1-\beta)^{\gamma+1}}{2\,\beta(\gamma+1)},$$

where $\mathfrak{e} = \frac{M_B^2 - M_p^2 + n_B^2 m_\pi^2}{2\,M_B^2 n_B}$ is the average fractional energy in the rest system of the isobar, carried away by a pion, and β is its velocity.

After eliminating the unknown factor η'^{γ} between eq. (V.2) and (V.3) one obtains a relation between

the isobar mass,	M_B,
the probability of its excitation,	s,
and the number of decay mesons,	n_B.

This relation is rather insensitive to the value of M_B, which cannot therefore be determined accurately from the available data. Therefore, we have plotted the relation in Figs. 7 a, b for two extreme values of M_B:

$$M_B = \infty \text{ and}$$

$$M_B = (M_B)_{\min}.$$

$(M_B)_{\min}$ is defined by

$$\frac{(M_B)_{\min}}{M_p} = 0.288\, n_B + \sqrt{1 + 0.06\, n_B^2}; \qquad \text{(V.4)}$$

it is the lowest possible mass for an isobar which decays into n_B pions and gives to each just the energy which a pion would receive in the deexcitation

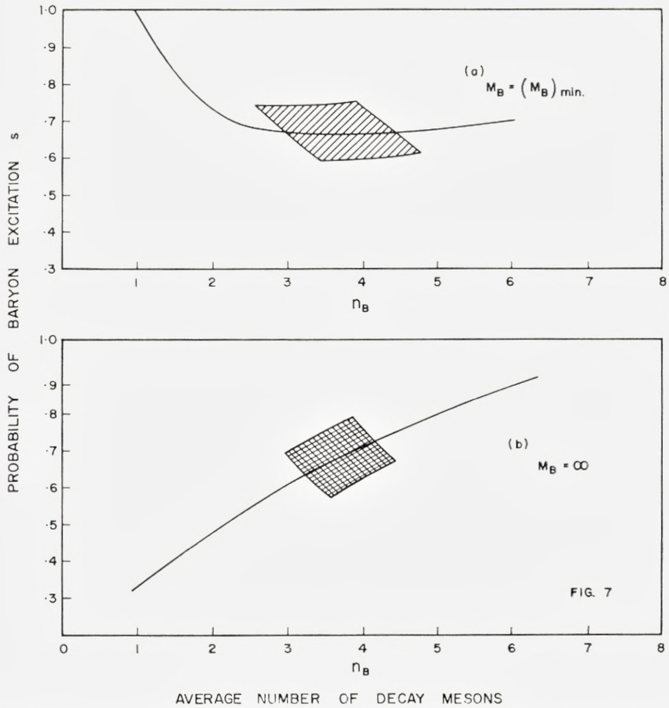

Fig. 7. The probability of baryon excitation, s, is plotted against the average number of decay mesons n_B. The shaded area indicates the values of s and n_B which are compatible with the observed ratios between the fluxes of primary nucleons, sea level protons, and sea level muons and with the observed energy dependence of multiplicity of meson production in high energy collisions (see Section V). Fig. 7a refers to a baryon with minimum mass as defined by eq. V.4; Fig. 7b refers to a baryon of infinite mass.

of the (3,3) resonance, i. e. the lowest excited state of the pion nucleon system.

The shaded area in Fig. 7 a indicates the range of values of s and n which satisfy the cosmic ray data

$$s = 0.70 \pm .07, \tag{V.5}$$

$$n_B = 3.5 \pm 0.5. \tag{V.6}$$

The vertical spread of the area of uncertainty in Fig. 7, i. e. the error in excitation probability, reflects mainly the uncertainty in the value of the nuclear interaction mean free path λ; the horizontal spread (the error in n_B) reflects mainly the present uncertainty in the mean multiplicity of created particles in the 1000 GeV region.

Fig. 7b ($M_B = \infty$) yields values for s and n_B which are not very different from those given in Fig. 7a, but it corresponds of course to an unrealistic assumption.

With $n_B = 3.5$, eq. (V.4) gives a lower limit for the average excited baryon mass $(M_B)_{\min} = 2300$ MeV. An indication that the actual value lies close to this limit is provided by the transverse momentum distribution of pions; the highest transverse momentum from an isobar of mass 2300 MeV occurs when it decays directly to the nucleon ground state by emission of a single pion; this value is 900 MeV/c and is very close to the upper limit of the observed transverse momentum distribution of neutral pions (16).

The fractional energy $(1-\eta')$ given to pionization can now be determined from eq. (V.2) or (V.3); it is (25 ± 7) %. This is not inconsistent with the observed multiplicity and the C-system energy of particles evaporated from the fireball if one also takes into account that ca. 20 % of the particles are heavier than pions (i. e. kaons or nucleon-antinucleon pairs).†

It must be remembered, however, that η' as determined here, and also the multiplicity relation which we have used, refer to targets consisting of light nuclei, not of single nucleons. Similarly, the value deduced for the excitation probability, s, refers to collisions of nucleons with light nuclei. The fractional energy loss to pionization in air $(1-\eta') \approx 25$ % should correspond to an energy loss of about 18 % for nucleon-nucleon collisions.

The fraction of the incident energy which goes into the pionization process is seen to be about half of the total energy loss of nucleons; because this energy is distributed among many particles, while in isobar deexcitation the energy is concentrated on a small number of pions, it is the latter which completely dominate the secondary cosmic radiation (see Fig. 1).

V.2. *The Composition of Isobar Decay Products*

a) The Ratio of Positive to Negative Pions and the Branching Ratio for K-Decay.

The positive excess $\langle\delta_\pi\rangle$ among decay pions of the forward isobars created by proton collisions with a charge symmetric target, as well as the

† If pions were the only particles evaporated from the fireball, the model predicts: $(1-\eta') = \dfrac{\varepsilon_F \gamma_C n_0 E^{\frac{1}{2}}}{E}$ (i. e. ≈ 11 %, if ε_F, which is the average pion energy in the C-system, is taken as $\varepsilon_F = 460$ MeV). But if only 80 % of the created particles are pions and 20 % are nucleons and antinucleons created with energy $\varepsilon \sim 3\,M_p$, the corresponding value of $(1-\eta')$ lies above 20 %.

branching ratio b_k for the decay mode $N^* \to K+Y$, can be obtained from the observed muon charge ratio as a function of energy. Inserting the experimental values of quantities entering into eq. (III.15), the ratio μ^+/μ^- has been plotted as a function of energy in Fig. 8 for various values of b_k. The experimental points shown are those of reference 17; above 100 GeV, this includes all measurements in the vertical direction. All other experimental data (ref. 18) agree within errors, but cover only the lower part of the energy spectrum. (One experimental point [18c] which is in disagreement with the others has been indicated in the figure).

Each of the curves in Fig. 8 corresponds to a definite value of the branching ratio b_k for the decay $N^* \to K+Y$ and to a definite value for the charge excess $\langle \delta_\pi \rangle$ amongst pions. On each curve there is also shown the value of δ'_π, which represents the charge excess for $(1-b_k)$ decays which do not involve strange particles. It is related to δ_π by

$$\delta'_\pi = \delta_\pi + c\, b_k,$$

where $c \approx 0.8$ depends slightly on the particular hyperon states involved.

While the data seem to indicate an energy dependence of the form given by eq. (III.15), they do not rule out the value zero for b_k.

We take

$$b_k = (10 \pm 10)\ {}^0/_0$$

and, correspondingly,

$$\frac{1}{\delta'_\pi} = 6.4 \begin{array}{l} +4.6 \\ -1.3 \end{array}.$$

(The highest value of the charge excess, $\frac{1}{\delta'_\pi} = 5.1$, corresponds to $b_k = 0$).

A value of $b_k \approx 10\ {}^0/_0$ corresponds to a kaon to pion production ratio

$$\left(\frac{K}{\pi}\right)_{\text{total}} = \frac{2\, b_k}{\frac{2}{3}(1-b_k)} \frac{B_k}{B} \approx 16\ {}^0/_0$$

(an upper limit on the production ratio $K_{\mu_2}/(\pi^+ + \pi^-) \approx 40\ {}^0/_0$ at about 70 GeV, corresponding approximately to $\left(\frac{K}{\pi}\right)_{\text{total}} \approx 80\ {}^0/_0$ was obtained by Ashton and Wolfendale[19] using the variation of muon flux with zenith angle).

b) The Ratio of Hyperons to Nucleons.

From the branching ratio $b_K = 0.10 \pm 0.10$ for the isobar decay mode $N^* \to K+Y$ one obtains the ratio of hyperons to nucleons among the forward emitted baryons in high energy collisions:

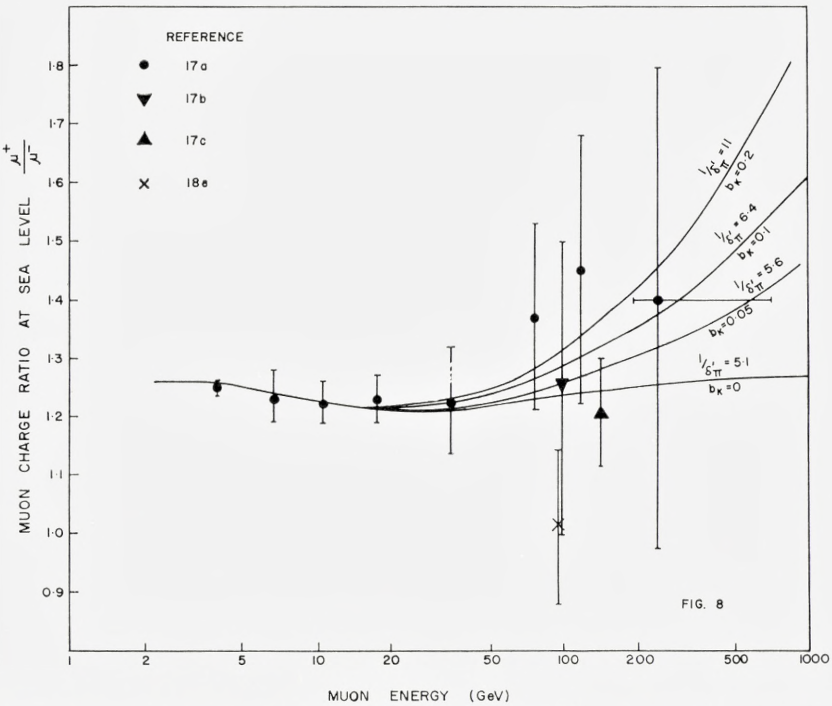

Fig. 8. The ratio of positive to negative muons at sea level[17] as a function of their energy. The relation given by eq. (III.15) has been drawn for various pairs of values $\langle\delta'_\pi\rangle$ and b_k consistent with the experimental data. [$\langle\delta'_\pi\rangle$ is the charge excess among pions from the isobar decays which do not involve strange particles, and b_k is the branching ratio for the decay mode $N^* \rightarrow K + Y$.]

$$\frac{Y}{N} = s b_K \approx 7\ {}^0\!/_0$$

and an estimate for the hyperon production cross section in high energy collisions:

$$\sigma_Y = 2\, s b_K\, \sigma_{\text{inelastic}} \approx 4.5\,\text{mb}.$$

However, because of the large experimental error which is still attached to the charge ratio of muons at high energy, one can place at present only an upper limit of 9 mb. on this cross section. (An earlier suggestion by one of the authors[4a], that this ratio may be high, is not borne out by this analysis). Fig. 8 shows that it is necessary to measure the ratio μ^+/μ^- at energy ~ 250 GeV to better than $5\ {}^0\!/_0$ in order to determine this cross section with an accuracy of $\sim 30\ {}^0\!/_0$.

VI. Discussion

It has been shown that the model of meson production which has been adopted permits a rigorous calculation of the intensity, composition, and energy distribution of secondary cosmic ray particles in terms of those of the primary radiation. The model contains only one fundamental (and plausible) assumption, namely, that the excitation of baryon isobaric states, which is a prominent feature of nuclear collisions at laboratory energies, remains of some importance for collisions at higher energies. The comparative unimportance in the secondary cosmic radiation of mesons produced with low energies in the center-of-mass system of nucleon-nucleon collisions follows directly from this basic assumption (see Appendix A and Fig. 1). The contribution of mesons produced in pion-nucleon collisions is unimportant when calculating the muon flux, but affects the ratio of pions to nucleons in the lower atmosphere. The derivation of expressions for the spectra of various secondary components is then straightforward and can be carried out for an arbitrary set of baryon states and an arbitrary combination of excitation probabilities and decay modes.

Making use of existing experimental data on secondary components one obtains then fairly definite values for the excitation probability of some "average" baryon state (~ 0.7) and for its mean mass (~ 2200 MeV) (although arguments against a higher value cannot be considered entirely conclusive). One obtains also an average value for the number of mesons emitted in the decay of the isobar ($n_B \approx 3.5$) and for their net charge ($|\pi^+ - \pi^-| \approx 0.35$). The branching ratio for the decay of the "average isobar" in the mode $N^* \rightarrow K + Y$ can be determined only roughly $b_k = 10\ (\pm 10)$ Finally, one can break down the average fractional energy loss suffered by nucleons in high energy collisions into a part given to the decay mesons ($\sim 25\ {}^0/_0$) and a part spent in creating particles in the C-system of nucleon-nucleon collisions (15–$20\ {}^0/_0$).

On the other hand, it does not seem possible at present to deduce in a unique way, solely from the average properties of the secondary cosmic radiation, the excitation probabilities and decay schemes of the individual baryon states involved. This seems feasible only, either by extrapolation from the lower energy region accessible to accelerators, or by studying individual high energy events, jets or airshowers, with the particular aim of obtaining data on the very fast or the very slow particles created in the interaction.

The assumption that, in nuclear collisions at all energies above ~ 10 GeV, isobars are produced with masses and decay properties very similar to those needed to reproduce the pion production spectra at 22 GeV [3], is sufficient to account for the existing observations on secondary cosmic rays in the atmosphere. It is consistent with existing data on particle production in high energy laboratories and with observations on jets. More complicated models to account for secondary cosmic ray particles are clearly possible; the very simple hypothesis which has been explored in this paper is not unique, but it seems to be adequate at this stage.

An application of the model discussed in this paper to the structure of airshowers will be the subject of a later study. It seems worth mentioning, however, that, if the mechanism described here remains essentially valid also in the energy region responsible for airshowers, the probability that an incident nucleon loses all but 25 % of its energy to neutral pions in a single collision will not be small. Such events will give rise to a few percent of air showers with abnormally high electron to muon ratios, i. e. with properties not unlike those of γ-ray induced airshowers whose possible occurrence and frequency is now under active investigation in various parts of the world. [20]

In view of the fact that particles created with low energy in the C-system of nucleon-nucleon collisions contribute little to the secondary cosmic ray flux (Appendix A) and that the recoil nucleons of terrestrial origin contribute only to the non-relativistic region of the energy spectrum (Appendix D), the high energy cosmic ray nucleons at any depth in the atmosphere represent essentially a sample of the extraterrestrial matter brought in by the incident cosmic radiation. Therefore, it is possible to study the fundamental question of whether the very high energy primary cosmic ray particles contain an appreciable fraction of anti-matter. If the primaries contained anti-nucleons, the nucleon spectrum on the ground above a few GeV should contain a corresponding fraction. By analysing the charge composition of the nucleon component at sea level upto ~60 GeV, it seems possible to investigate a possible fraction of antimatter in the primary radiation upto energies of order 10,000 GeV/nucleon.

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Appendix A

Experimental Justification for the Choice of the Model

It will be shown in this Appendix that each of the assumptions, on which the calculations in this paper are based, either follows from, or is at least consistent with, all experimentally well established facts regarding high energy collisions and also that once the not too infrequent occurrence of excited baryon states is admitted, one is led necessarily to the conclusion that the secondary cosmic radiation is dominated by the decay products of these baryon isobars.

The basic features of the model are:

1) A fireball is created at rest in the C-system of a nucleon-nucleon collision.*

2) The fireball evaporates, giving rise to n_F mesons, with an isotropic or moderately anisotropic angular distribution in its rest system. The ratio of pions to non-pions among the created particles does not change with energy.

3) In the evaporation process each of these mesons receives a momentum whose average value

$$\overline{P}_F = \frac{4}{\pi}\overline{P}_\perp \sim 450 \text{ MeV/c},$$

and whose maximum value

$$(P_F)_{\max} \approx 800 \text{ MeV/c}.$$

Since the average transverse momentum, $\overline{P}_\perp$, is known to be independent of energy, $\overline{P}_F$ also is independent of energy.

4) The number n_F of mesons evaporated from the fireball increases in proportion to the energy available in the C-system of the nucleon-nucleon collision.

5) The incident baryon emerges in some excited state with a probability, s, and decays by emitting on the average n_B pions, whose momentum in the baryon rest system is P_B.

Taken together, conditions 1) to 4) imply also that the incident baryon transfers on the average a constant fraction $(1-\eta')$ of its energy to the fireball.

* The terms "fireball" and "pionization" were first introduced by G. Cocconi.

A.1 *The Pionization Process*

a) Energy spectra of mesons.

The energy distribution of mesons in the C-system of a nucleon-nucleon collision can be measured reliably provided the following conditions are satisfied:

α) The event occurs within or in the vicinity of the detector.
β) The energy of the incident particle is known with adequate accuracy.
γ) Observability and measurability are not functions of particle momentum, at least within a broad and well defined momentum range.

The only experiments carried out so far which satisfy all these conditions are those of the Moscow group [5a]. Here, the incident energy is measured by a total absorption calorimeter and the momenta of secondary charged particles by the curvature of their tracks in a magnet cloud chamber. They obtain $\overline{P}_F = 450$ MeV/c. Unfortunately, the experiments are limited to incident energies below 500 GeV.

If one relaxes the second condition and accepts also those experiments in which the incident energy is not measured but deduced from symmetry arguments and from the angular distribution of shower particles, one admits three other classes of experiments:

i) Emulsion experiments in which geometrically favourable events are selected and the momenta of all shower particles of a given collision are determined by scattering measurements. Experiments of this type, carried out by Jain[5c], are discussed in ref. 1; the average momentum of particles belonging to the pionization process is $\overline{P}_F = 430$ MeV/c. Measurements of Schein et al.[5b], analysed in an analogous manner, yield a somewhat higher value $\overline{P}_F = 600$ MeV/c. All the measurements refer to incident energies of order 2500 GeV.

ii) Cloud chamber experiments with magnetic field but without calorimeter. The experiment of Hansen and Fretter[5d] yields $\overline{P}_F = 470$ MeV at 100 GeV and $\overline{P}_F = 370$ MeV at 1000 GeV primary energy; that of Montanet et al.[5e] yields $\overline{P}_F = 410$ MeV/c at 100 GeV.

iii) Cloud chambers containing enough absorbers both of low and of high atomic weight, so that all γ-rays are converted and observable and the energy of neutral pions produced in the interaction can be estimated from the ensuing showers. Such an experiment is that of Lal et al.[5f]

which yielded $\bar{P}_F = 430$ MeV/c. This experiment refers to a range of incident energies 20 GeV $\leqslant E \leqslant$ 150 GeV.

One sees that the experimental results on C-system momenta agree fairly well with each other and suggest the adopted value $\bar{P}_F \approx 450$ MeV/c.

The average transverse momentum of shower particles has been measured by many experimenters. Most particles have transverse momenta $P_\perp > m_\pi c$, i. e. they are relativistic in the C-system; the generally accepted value of $\bar{P}_\perp$ is independent of the incident energy and lies between 350 and 400 MeV/c. Thus, assumption 3), that $\bar{P}_f$ is energy independent and about 20–30 $^0/_0$ larger than the average transverse momentum of shower particles, is in conformity with existing measurements.

b) Angular Distribution of Mesons.

The ratio of average total to average transverse momentum of shower particles in the C-system suggests a fairly isotropic angular distribution for the particles created in the pionization process. Direct measurements of the angular distribution of shower particles in the L-system support this conclusion.

Since the transverse momentum distribution shows that most particles are relativistic in the C-system, one may use, when transforming to the L-system, the approximation

$$\beta_c/\beta^* \approx 1.$$

With this approximation a fireball which emits particles with an angular distribution proportional to $\cos\theta^n$ produces an angular distribution in the L-system

$$N(x)\,dx = \tfrac{1}{2}\,d\left(\tanh^{n+1}(x + \log\gamma_c)\right), \tag{A.1}$$

where γ_c is the C-system energy of the incident nucleon in rest mass units and $x = \log\tan\theta$.

This distribution, when plotted against x, has two maxima:

$$x_m = -\log\left\{\gamma_c\left[\gamma_F \pm \sqrt{\gamma_F^{\,2} - 1}\right]\right\}, \tag{A.2}$$

where $\gamma_F = \sqrt{1 + \frac{n}{2}}$. In the case of complete isotropy, ($n = 0$), the maxima coalesce into a single maximum at $x_m = -\log\gamma_c$, and one obtains a quasi-gaussian distribution with a root mean square deviation $\sigma = 0.39$. Normally, however, fireballs must be expected to have intrinsic angular momentum, so that a certain measure of anisotropy in the evaporation and therefore double maxima in the log tan θ distribution should be common; they should become somewhat more pronounced with increasing energy. In many showers only a single hump can be seen, but when two maxima can be resolved, separations x_m are typically in the range $\frac{1}{2} < x_m < 2$, corresponding to C-system angular distributions between cos $\theta^{0.125}$ and cos $\theta^{2.5}$. The observed L-system angular distribution can therefore be interpreted as due to moderately anisotropic emission from a spinning fireball at rest; alternatively, (as one can see from eq. A.2) it can also be interpreted as being due to emission from two spinless fireballs moving parallel to the colliding nucleons with velocities in the C-system $\beta_F = \sqrt{\frac{n}{n+2}}$ and a fairly symmetric distribution for each hypothetical emission centre

$$\frac{\text{forward} - \text{backward}}{\text{forward} + \text{backward}} = 1 - 2\left(\frac{n}{n+2}\right)^{\frac{n+1}{2}} \approx 25 - 30\,\%.$$

The latter interpretation has been stressed especially by the Krakow group[(21)].

An upper limit for $(P_F)_{\max} \approx 800$ MeV/c is dictated by the observed absence or rarity of transverse momenta higher than this value[(16)].

Thus the assumptions 1), 2), and 3), i.e. creation of a fireball at rest emitting relativistic particles fairly isotropically, are in accord with experiments.

It is possible that fireballs have discrete mass values of the order of 2 GeV, as suggested by Hasegawa[(22)], which may account for their absence or rarity in $p-p$ collisions at accelerator energies and for the large fluctuations observed in the angular distributions in the energy range of 100–300 GeV[(5 a)].

A.2 *Relation between Multiplicity and Energy.*

Only few measurements of multiplicity exist where the energy of the incident nucleon is high and measurable and where at the same time interactions have been collected without strong bias against low multiplicity events. Below 30 GeV one has quite accurate results based on accelerator data. At 70,100, 300, and 1000 GeV one has cloud chamber data[(5a, d, f)]. The measured mean multiplicities may be slightly too high, because of some residual bias against very low multiplicities[(5d)].

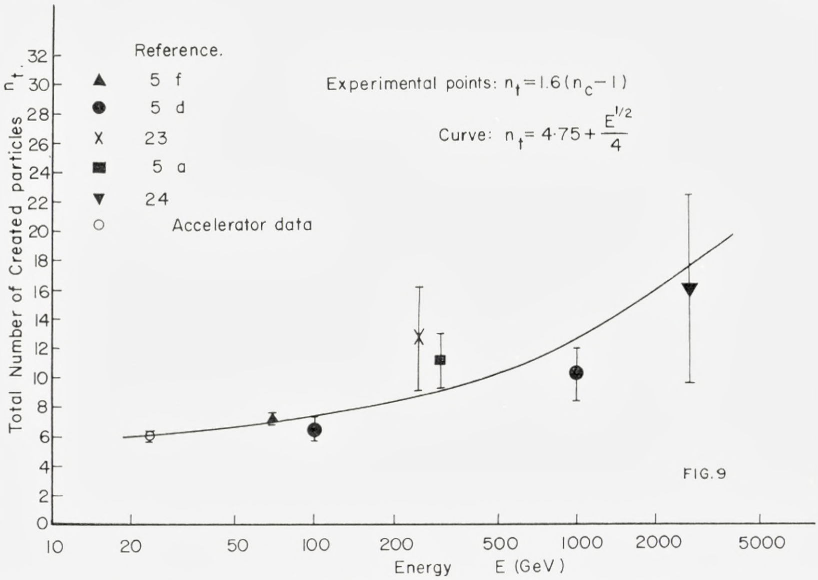

Fig. 9. The total number of created particles (charged and neutral), which on the average are emitted in a collision between a nucleon and a target nucleus of low atomic weight, is plotted as a function of the energy of the incident nucleon. The curve represents relation A.4.

At 250 GeV and at 2700 GeV, fairly reliable though statistically poor measurements of the multiplicity of charged particles were obtained by Lohrmann et al.[23] and by Abraham et al.[24] These authors used heavy primary nuclei whose energy per nucleon can be determined by various well-known methods; they then searched for interactions by scanning along the tracks of singly charged break-up products, i.e. fragments of the incident nucleus. One source of error could be the presence of deuterons or tritons among the break-up products, and another, a possible inclusion of interactions with heavy nuclei, silver or bromine, even when only events with less than five heavy ionizing particles are accepted. Both types of error may lead to some overestimate of multiplicity.

In Fig. 9 the total number of created particles, n_t, at various energies is shown; n_t has been calculated from the measured number of charged particles, n_c, using the relation

$$n_t = \left[0.8\left(\frac{3}{2}\right) + 0.2\,(2)\right] \cdot (n_c - 1) = 1.6\,(n_c - 1). \qquad \text{(A.3)}$$

The first term in eq. (A.3) refers to pions and the second to particles of isospin $\frac{1}{2}$. The relation (A.3) is energy independent, because the ratio of non-pions to pions among created particles is known to be independent of energy.[(25)]

The model requires a multiplicity relation of the form $n_t = 2\,sn_B + n_0 E^{\varrho}$, where s is the probability of baryon excitation, n_B the average number of mesons emitted in their decay, and $n_0 E^{\varrho}$ represents the creation of mesons in the pionization process.

The available data are neither numerous nor accurate enough to determine the form for the multiplicity law uniquely. A lead is provided by the approximate invariance of the nucleon energy spectrum with atmospheric depth, which suggests that the average energy loss of nucleons in collisions with nitrogen is energy independent. In this simplest case, the multiplicity relation must be of the form

$$n_t = 2\,sn_B + n_0\,E^{1/2}. \qquad \text{(A.4)}$$

The curve in Fig. 9 represents eq. (A.4). A fit with experiment requires the following values for the parameters:

$$2\,sn_B = 4.75 \pm 0.25$$

$$n_0 = 1/4$$

i.e.

$$n_t = 4.75 + \frac{E^{1/2}}{4}\,(E \text{ is in GeV}).$$

This relation between energy and number of created particles is consistent with existing measurements. (Although it cannot be deduced from the data in a unique manner, the data also do not warrant as yet the assumption of a more complicated relationship).

The multiplicity relation for high energy pions is still less certain than that for nucleons.

In the text (eq. IV.1 a), the relation

$$\nu_t = sn_B + \nu_0\,E_{\mathrm{GeV}}^{1/2}, \qquad \text{(IV. 1 a)}$$

which corresponds to a high degree of inelasticity, has been used as yielding sufficient multiplication of pions in the lower atmosphere to reproduce the measured pion/proton ratio near sea level[(14)]. Agreement can be obtained for $\nu_0 = 0.7$.

Using again $sn_B \approx 2.4$, the formula gives a reasonable multiplicity of created particles for $\pi - p$ collisions from 8 to 800 GeV, the energy region in which measurements are available. Apart from the term due to baryon isobars, the two multiplicity laws $n_t(E)$ for nucleons (A.4) and $\nu_t(E)$ for pions (IV.1) are then related by

$$n_t(E) = \nu_t\left(E\frac{m_\pi}{M_p}\right),$$

i. e. the pionization in nucleon-nucleon collisions behaves as if it were due to completely inelastic collisions between two target masses, not very different from those of free pions; in that case,

$$\frac{n_0}{\nu_0} = \sqrt{\frac{m_\pi}{M_p}} = 0.39, \tag{A.5}$$

and the fractional energy going to pionization in nucleon-nucleon collisions is

$$1 - \eta' = \frac{m_\pi}{M_p} \sim 15\ {}^0/_0,$$

values which are rather similar to those arrived at in Section IV.

If such collisions lead to boson quanta of rest mass $M_F \approx 2\,M_p$, as suggested by HASEGAWA(22), the creation of particles by pionization in nucleon-nucleon collisions will become important only if $m_\pi \gamma_c \approx M_p$ or $E \geqslant 100$ GeV; this may be connected with the smallness of the isotropic low energy pion component in the C-system of nucleon-nucleon collisions at accelerator energies(3).

A.3 *Relative Contributions to the Secondary Cosmic Ray Flux of Mesons from Isobar Decay and Mesons from Pionization.*

The two different processes by which particles are created in this model have now been specified sufficiently well so that their relative contribution to pion production in the atmosphere can be evaluated reliably and in a straightforward manner.

If the nucleon energy spectrum is of the form $dE/E^{\gamma+1}$, the spectrum of pions produced in the pionization process will be given by

$$\left.\begin{aligned} g_F(E)\,dE &= n_0\left(\frac{\varepsilon_F^2}{2\,M_p}\right)^{\gamma-\varrho}\frac{(1+\beta_F)^{2(\gamma+\frac{1}{2}-\varrho)}(1-\beta_F)^{2(\gamma+\frac{1}{2}-\varrho)}}{2\,\beta_F(\gamma+\frac{1}{2}-\varrho)}\frac{dE}{(E^2)^{\gamma+\frac{1}{2}-\varrho}} \\ &\approx \frac{n_0\,dE}{(\gamma+\frac{1}{2}-\varrho)(E^2)^{\gamma+\frac{1}{2}-\varrho}}\left(\frac{2\,\varepsilon_F^2}{M_p}\right)^{\gamma-\varrho}. \end{aligned}\right\} \tag{A.6}$$

ε_F is the energy of the mesons in the rest system of the fireball and β_F is their velocity.

This formula applies to isotropic emission. In the case of extreme anisotropy, where half the particles are emitted at 0° and half at 180°, g_F must be multiplied by $(\gamma + \frac{1}{2} - \varrho) \simeq 1.67$. An anisotropic emission of particles from the fireball does not, therefore, invalidate the conclusions derived in this section.

The spectrum of pions produced in isobar decay is given by

$$\left.\begin{aligned} g_B(E)\,dE &= sn_B \left(\frac{\eta'\,\varepsilon_B}{M_B}\right)^{\gamma} \frac{(1+\beta_B)^{\gamma+1} - (1-\beta_B)^{\gamma+1}}{2\,\beta_B(\gamma+1)} \frac{dE}{E^{\gamma+1}} \\ &\approx \frac{sn_B\,dE}{(\gamma+1)\,E^{\gamma+1}} \left(\frac{2\,\eta'\,\varepsilon_B}{M_B}\right)^{\gamma}, \end{aligned}\right\} \quad (A.7)$$

where ε_B is the energy of decay pions in the rest system of the baryon isobar of mass M_B and β_B is their velocity.

For the lowest lying isobar, (the $T = 3/2$, $J = 3/2$ state),
$\varepsilon_B \approx 270$ MeV and it is probably higher for heavier isobars.
$\varepsilon_F \approx 470$ MeV, as shown in experiments [(5)] discussed earlier.
η' may be taken to be approximately constant and equal to 0.8 (see Section IV).

Assuming an exponent $\gamma = 1.67$ for the primary energy spectrum, the relative contribution of the two processes, $\frac{g_B}{g_F}$, as a function of pion energy has been plotted in Fig. 1 for various assumed values of the product sn_B (sn_B is the average number of pions per collision from the deexcitation of the forward isobar). The constants n_0 and ϱ which characterize the size of the fireball have been chosen such that the total multiplicity of created particles n_t agrees with the experimental data for targets of low atomic number and for incident nucleon energies of 30 GeV ($n_t = 6$) and of 2700 GeV ($n_t \approx 18$). The appropriate multiplicity relation is shown on each of the curves in Fig. 1.

From these curves one sees that isobar pions dominate in the atmosphere at all energies if one accepts the value $sn_B \approx 2$; they dominate above 10 GeV for values of sn_B as small as 0.5. The dominance of the isobar decay process as a contributor to the flux of secondary cosmic ray particles increases rather rapidly with energy; of course this holds not only for mesons but also for their decay products, i.e. for muons, γ-rays, neutrinos etc. *Thus,* *unless baryon excitation is much less frequent in high energy collisions than it is at laboratory energies, the secondary cosmic radiation consists essentially of the decay products of baryon isobars and their progeny.*

Appendix B

Formulae for the General Case of an Arbitrary Set of Isobaric States and for Arbitrary Decay Schemes

The simple model on which this paper is based permits a fairly rigorous calculation of nucleon and meson intensities and spectra at all points in the atmosphere.

We derive here formulae for the nucleon flux, the pion production spectrum, and the charge composition of nucleons and of pions, for the general case of an arbitrary set of baryon isobars, each with its own excitation probability and with arbitrary decay chains. Although the formulae involve summations over running indices, they can be evaluated easily for specific cases.

B.1 *The Nucleon Flux.*

Let the isobar of type r be produced with an interaction length λ_r and let it carry a fraction η_r' of the energy of the incident nucleon. The physical isobaric states are numbered 0,1,2,3 . . . r . . . in ascending order of mass, so that 0 denotes the nucleon ground state. Direct transition between any two of these states is assumed to lead to emission of a single boson which we shall take to be a pion, but which may equally well be a boson isobar, which subsequently disintegrates into pions.

A collision of type i is defined by specifying the isobar which is produced as well as the specific chain, σ, by which it decays into a nucleon and a number of pions. Taking account of the Poisson fluctuations in the number of collisions of each type, a simple extension of the argument given in chapter II (eq. II.4) shows that the attenuation length of nucleons in the atmosphere is given by the relation

$$\frac{1}{\Lambda} = \frac{1}{\lambda}(1 - \langle \eta^{\gamma} \rangle), \tag{B.1}$$

where

$$\langle \eta^{\gamma} \rangle = \sum_{i=1}^{\infty} \frac{\lambda}{\lambda_i} \eta_i^{\gamma}, \tag{B.2}$$

$$\frac{1}{\lambda} = \sum_{i=0}^{\infty} \frac{1}{\lambda_i}, \tag{B.3}$$

and η_i is the fraction of the incident energy retained by the nucleon in the i'th type of collision.

The problem is to express $\langle \eta^\gamma \rangle$ in terms of the mass and decay modes of the isobars. The averaging process may be broken up into three independent parts.

α)

For a particular isobar r and for decay to the ground state via a definite set of intermediate states $\sigma = (\sigma_1, \sigma_2, \sigma_3, \ldots)$ one can average over the angular distributions of decay and obtains

$$\langle \eta^\gamma \rangle_{r,\sigma} = \eta_r'^{\gamma} A_{r,\sigma_1} A_{\sigma_1,\sigma_2} A_{\sigma_2,\sigma_3} \ldots A_{\sigma_n,0} \tag{B.4}$$

$$A_{l,m} = \mathfrak{E}_{l,m} \frac{(1+\beta'_{l,m})^{\gamma+1} - (1-\beta'_{l,m})^{\gamma+1}}{2\,\beta'_{l,m}(\gamma+1)} \tag{B.5}$$

and $\mathfrak{E}_{l,m}$ is the fractional energy in the rest system of isobar "l" which is carried away by isobar "m", and $\beta'_{l,m}$ is its velocity.*

β)

The average over different decay modes involving different intermediate states σ is obtained by summing all possible expressions of the type

$$(bA)_{r,\sigma_1} (bA)_{\sigma_1,\sigma_2} (bA)_{\sigma_2,\sigma_3} \ldots (bA)_{\sigma_n,0} = Y_{r,\sigma}, \tag{B.6}$$

where $b_{l,m}$ is the branching ratio for transition from the state "l" to the state "m". The various terms in $Y_{r,\sigma}$ are subject to the restriction $\sigma_j < \sigma_{j-1}$.

The result of summing over all possible decay chains σ is designated by

$$Y_r = \sum_\sigma Y_{r,\sigma}. \tag{B.7}$$

One obtains

$$\eta_r^\gamma = \eta_r'^{\gamma} Y_r. \tag{B.8}$$

* The expression for $A_{l,m}$ is given here for isotropic decay; it is easily calculated for specified non-isotropic emission of decay products.

γ)

As the final step one must average over contributions from different isobars which have to be weighted according to excitation probability, e.g. with a weight factor which is inversely proportional to the mean free path for exciting the particular baryon state

$$\langle \eta^{\gamma} \rangle = \sum_{r=0}^{\infty} \frac{\lambda}{\lambda_r} \eta_r^{\gamma} = \sum_{r=0}^{\infty} \frac{\lambda}{\lambda_r} \eta_r'^{\gamma} Y_r. \quad \text{(B.9)}$$

B.2 *The Nucleon Charge Ratio.*

In Section III it is shown that the charge composition of nucleons at a depth x can be written as

$$\left(\frac{p-n}{p+n}\right)_x = \delta_x = \delta_0 \, e^{-(x/\lambda) \langle 2 \eta^{\gamma} w \rangle}. \quad \text{(III.2)}$$

The average $\langle \eta^{\gamma} w \rangle$ may now be written as

$$\langle \eta^{\gamma} w \rangle = \sum_{r=0}^{\infty} \frac{\lambda}{\lambda_r} \eta_r'^{\gamma} \sum_{\sigma} w_{r,\sigma} Y_{r,\sigma}, \quad \text{(B.10)}$$

where $w_{r,\sigma}$ is the probability of producing a neutron from the decay of an isobar r, if the isobar is produced by an incident proton and decays through a particular decay chain σ. An explicit expression for $w_{r,\sigma}$ in terms of branching ratios is given later (in eq. B.19).

The proton and neutron spectra in the atmosphere are given by eq. (III.5).

B.3 *The Production Spectrum of Pions due to Isobar Decay.*

If ε is the fractional energy in the isobar rest system, carried away by a pion, and $n^{\pm}$ is the number of positive or negative pions emitted in the decay of an isobar produced in a collision of a proton with an air nucleus, then the production spectrum of charge pions at a depth x is given by

$$P_{\pi\pm}(x, E)\, dx = \frac{dx}{\lambda} \frac{N(0, E)}{2} e^{-x/\Lambda} \left[\langle (n^+ + n^-)(\eta' \varepsilon)^{\gamma} \rangle \pm \delta x \langle (n^+ - n^-)(\eta' \varepsilon)^{\gamma} \rangle \right]. \quad \text{(B.11)}$$

The problem is to find an explicit expression for $\langle n^{\pm} (\eta' \varepsilon)^{\gamma} \rangle$ in terms of isobar masses and their excitation probabilities and decay branching ratios.

Starting with any isobar r, a pion may be obtained from any transition

$l \to m$, $(m < r)$. The state l itself, however, is in general the result of a series of decay combinations of states intermediate between r and l. Thus, in order to evaluate $\langle n^{\pm}(\eta' \varepsilon)^{\gamma} \rangle$, one must first sum over all possible decay schemes which can lead from the isobar r to the isobar l.

α)

$$\left. \begin{aligned} [n^{\pm}(\eta' \varepsilon)^{\gamma}]_{r,l,m} = \eta_r'^{\gamma} \alpha_{l,m} b_{l,m} \sum_{\sigma} V_{r,\sigma}^{(\pm)l,m} (bA)_{r,\sigma_1} \\ \times (bA)_{\sigma_1,\sigma_2} (bA)_{\sigma_2,\sigma_3} \ldots (bA)_{\sigma_n,l} \end{aligned} \right\} \quad \text{(B.12)}$$

$\alpha_{l,m}$ is defined in analogy to $A_{l,m}$, i.e.

$$\alpha_{l,m} = \varepsilon_{l,m}^{\gamma} \frac{(1+\beta_{l,m})^{\gamma+1} - (1-\beta_{l,m})^{\gamma+1}}{2\beta_{l,m}(\gamma+1)}, \quad \text{(B.13)}$$

where $\varepsilon_{l,m}$ is the fractional energy in the rest system of isobar l, carried away by a pion in the transition to the baryon state m, and $\beta_{l,m}$ is its velocity. The b's are the various branching ratios; $V_{r,\sigma}^{(\pm)l,m}$ is a function of the isospins of the initial and intermediate isobaric states and has the following significance: If a proton is incident on a target consisting of an equal number of protons and neutrons and emerges in the excited baryon state r, and if then this state r decays by a particular chain σ to the intermediate state l, then $V_{r,\sigma}^{(\pm)l,m}$ expresses the probability that the pion from the transition $l \to m$ is positively or negatively charged.

So far we have averaged over the angles of decay and combinations of intermediate states, which can lead from isobar r to isobar l.

β) Next, one must sum over all possible states l and m.

$$\langle n^{\pm}(\eta' \varepsilon)^{\gamma} \rangle_r = \sum_{l=1}^{r} \sum_{m=0}^{l-1} \langle n^{\pm}(\eta' \varepsilon)^{\gamma} \rangle_{r,l,m}. \quad \text{(B.14)}$$

γ) Lastly, one averages over contributions from different isobars, r, weighted according to the relative excitation probabilities λ/λ_r and obtains

$$\langle n^{\pm}(\eta' \varepsilon)^{\gamma} \rangle = \sum_{r=0}^{\infty} \frac{\lambda}{\lambda_r} \langle n^{\pm}(\eta' \varepsilon)^{\gamma} \rangle_r$$

or explicitly

$$\left. \begin{aligned} \langle n^{\pm}(\eta' \varepsilon)^{\gamma} \rangle = \sum_{r=0}^{\infty} \frac{\lambda}{\lambda_r} \eta_r'^{\gamma} \sum_{l=1}^{r} \sum_{m=0}^{l=1} \alpha_{l,m} b_{l,m} \\ \sum_{\sigma} V_{r,\sigma}^{(\pm)l,m} (bA)_{r,\sigma_1} (bA)_{\sigma_1 \sigma_2} \ldots (bA)_{\sigma_n,l}. \end{aligned} \right\} \quad \text{(B.15)}$$

We define

$$\langle \delta_\pi \rangle = \frac{\langle n^+ (\eta' \varepsilon)^\gamma \rangle - \langle n^- (\eta' \varepsilon)^\gamma \rangle}{\langle n^+ (\eta' \varepsilon)^\gamma \rangle + \langle n^- (\eta' \varepsilon)^\gamma \rangle} \tag{B.16}$$

and

$$\langle B \rangle = \langle n^+ (\eta' \varepsilon)^\gamma \rangle + \langle n^- (\eta' \varepsilon)^\gamma \rangle . \tag{B.17}$$

(As shown in Section II, all secondary components of the cosmic radiation in the atmosphere are proportional to the parameter $\langle B \rangle$).

The charge composition of pions produced by the nucleon component at atmospheric depth x can be written as

$$\frac{P_{\pi^+} - P_{\pi^-}}{P_{\pi^+} + P_{\pi^-}} = \langle \delta_\pi \rangle \, \delta_x , \tag{III.5}$$

where δ_x gives the nucleon charge composition at that level (eq. III.2).

The total number of pions of either charge in the complete deexcitation of isobar r (produced in proton collisions) is given by

$$n_r^\pm = \sum_\sigma n_{r,\sigma}^\pm = \sum_\sigma \sum_{l=1}^{r} \sum_{m=0}^{l-1} b_{l,m} V^{(\pm)}{}_{r,\sigma}^{l,m} b_{r,\sigma_1} b_{\sigma_1,\sigma_2} \ldots b_{\sigma_n, l} . \tag{B.18}$$

For a neutron collision with a charge symmetric target the isospin functions transform as

$$V^{(+)} \rightarrow V^{(-)},$$

$$V^{(-)} \rightarrow V^{(+)}.$$

The quantity $w_{r,\sigma}$ appearing in eq. (B.10), i. e. the charge exchange probability for a nucleon excited to the state r and decaying via a particular mode σ, can now be expressed by

$$w_{r,\sigma} = q_r + \sum_{l=1}^{r} \sum_{m=0}^{l-1} b_{l,m} \left[V^{(+)}{}_{r,\sigma}^{l,m} - V^{(-)}{}_{r,\sigma}^{l,m} \right] b_{r,\sigma_1} b_{\sigma_1,\sigma_2} \ldots b_{\sigma_n, l} , \tag{B.19}$$

where q_r is the average charge difference between the incident proton and the resulting isobar r.

The expression for $\langle B \rangle$ (eqs. B.15, 17) can be greatly simplified in the special case that the isobar decays by cascade in small steps of comparable size such that $2\, m_\pi < (M_l - M_m = \Delta) \ll M_B$.

Then

$$\mathfrak{E}_{l,m} = \frac{M_l^2 + M_m^2 - m_\pi^2}{2\,M_l^2} \approx 1 \quad \text{and} \quad A_{l,m} \approx 1\,, \tag{B.20}$$

while

$$\left.\begin{aligned} \varepsilon_{l,m} &= \frac{M_l^2 - M_m^2 + m_\pi^2}{2\,M_l^2} \approx \frac{\Delta}{M_l}, \quad \text{and, therefore,} \\ \alpha_{l,m} &= \frac{\left(\frac{2\,\Delta}{M_l}\right)^\gamma}{\gamma+1}. \end{aligned}\right\} \tag{B.21}$$

Equations (B.15, 17) reduce then to

$$\langle B \rangle = \sum_{r=0}^{\infty} \frac{\lambda}{\lambda_r} \frac{[2\,(\eta'\,\varepsilon_r)]^\gamma}{(\gamma+1)} (n_r^+ + n_r^-), \tag{B.22}$$

where ε_r is the fractional energy in the isobar rest system carried away by a pion.

Appendix C

Kaon Production from Non-Strange Isobars and the Resulting Muon Flux

The kaon flux due to non-strange isobars can be calculated in a manner similar to the pion flux. (As explained in the text, the most prominent decay mode will be $N^* \to K + Y$ and therefore the flux of anti-kaons produced in this manner is expected to be small and will be neglected).

Since kaons do not multiply in subsequent interactions, it is only necessary to set $q_+ + q_- = 0$ in eq. (II.11) and replace the values for mass, lifetime, and interaction mean free path of pions by those of charged (i. e. positive) kaons. Apart from the factor denoting the branching ratio for this decay mode, one obtains the flux of charged kaons

$$F_k = \frac{S_0 \langle B_k \rangle x}{E^{\gamma+1}\,\lambda} e^{-\frac{x}{\lambda_k}} \sum_{i=0}^{\infty} \frac{\left(1 - \frac{\lambda_k}{\Lambda}\right)^i}{i!\,(1+i+u_k)}, \tag{C.1}$$

$$\text{where } u_k = \frac{h_0\, m_{k+}}{c\,\tau_{k+}\, E} = \frac{e_{k+}}{E} = \frac{945}{E_{\text{GeV}}}; \tag{C.2}$$

$\langle B_k \rangle$ is defined in analogy with $\langle B \rangle$ (eqs. B. 15, 17)

$$\langle B_k \rangle = \frac{1}{2} s\,(\eta' \varepsilon_k)^\gamma \frac{(1+\beta_k)^{\gamma+1} - (1-\beta_k)^{\gamma+1}}{2\,\beta_k\,(\gamma+1)}, \tag{C.3}$$

where $\varepsilon_k = \dfrac{M_B{}^2 - M_Y{}^2 + m_k{}^2}{2\,M_B{}^2}$ is the fractional energy in the rest frame of the isobar, carried away by the kaon, and β_k is its velocity.

For $\lambda_k \approx \Lambda$

$$F_k = \frac{S_0 \langle B_k \rangle}{E^\gamma (E + \mathrm{e}_{k+})} \frac{x}{\lambda} e^{-\frac{x}{\Lambda}}. \tag{C.3 a}$$

In analogy with the procedure in Section II.4 one obtains the production spectrum of muons

$$P_{\mu_k}(x, E) = 0.69 \frac{\mathrm{e}_{k+}}{E} \frac{F_k[x, (r_k E)]}{x}, \tag{C.4}$$

where 0.69 represents the fraction of muons which arise directly from kaon decay without an intermediate pion, i. e. mostly $k_{\mu 2}$ decays.

$$r_k = \frac{2}{1 + \left(\dfrac{m_\mu{}^2}{m_k}\right)} \left[\frac{2\,\beta\,(\sigma+1)}{(1+\beta)^{\sigma+1} - (1-\beta)^{\sigma+1}}\right]^{\frac{1}{\sigma}} \approx (\sigma+1)^{\frac{1}{\sigma}} = 1.7$$

(the "best fitting" exponent σ is defined by

$$\left.\begin{aligned} -(\sigma+1) = \frac{\partial \log F_k}{\partial \log E} &= \gamma + 1 + \frac{\mathrm{e}_{k+}}{E + \mathrm{e}_{k+}} \\ &\approx 3.2 \ \ (\text{for } E \approx 500 \text{ GeV}). \end{aligned}\right\} \tag{C.5}$$

In anaolgy with (II.17) the resulting muon flux is

$$F_{\mu_k}(x, E) = 0.69 \frac{S_0 \langle B_k \rangle}{\lambda} \frac{\mathrm{e}_{k+}}{r_k{}^{\gamma+1}} \left(\frac{\lambda_k E}{x}\right)^v \frac{\Gamma(v+1)}{E'^{\,\gamma+2+v}\left(1 + \dfrac{\mathrm{e}_{k+}}{r_k E'}\right)}, \tag{C.6}$$

where $E' = \mathrm{E} + b(x - \overline{x})$ as defined in (II.17).

Appendix D

Corrections at the Low Energy End of the Nucleon Spectrum

In its simplest form the model treated here assumes that the fractional energy loss of nucleons (which is about 45 $^0/_0$) does not depend on the collision energy; therefore the primary power law is preserved throughout the atmosphere. This assumption can be valid only as long as the energy lies sufficiently high above the threshold for producing the most prominent isobaric states (i. e. $\gtrsim$ 10 GeV). Below that energy, meson production drops and collisions tend to become more elastic. In this region the fractional loss of total energy becomes small, but the fractional loss of kinetic energy is known to remain of the order of 50 $^0/_0$. Thus, if the spectrum of secondary nucleons is written in the form

$$N\,dE \sim \frac{dE}{(E-M)^{\gamma+1}} \quad \text{rather than} \quad \frac{dE}{E^{\gamma+1}},$$

it will represent the nucleon flux down to lower values of the energy without appreciably changing the results obtained in the region of higher energies (eq. II.2). For this reason, the uncorrected nucleon spectrum is represented in Fig. 2a as a power law in kinetic, rather than total energy.

Before extrapolating the proton spectrum (eq. III.3) into the low energy region, there are two other corrections to be made:

a) Energy loss by ionization and
b) The production of recoil nucleons.

a) The loss of energy due to ionization.

The probability $p(y, x)$, that the average incident primary cosmic ray nucleon (charge composition δ_0) is a proton at depth y and also at the point of observation x, is given by

$$p\,(y,x) = \frac{1}{4}\left\{1 + \delta_0\, e^{-\frac{2\,xw}{\lambda}} + \delta_0\, e^{-\frac{2\,yw}{\lambda}} + e^{-\frac{2\,(x-y)\,w}{\lambda}}\right\}, \tag{D.1}$$

where W is the charge exchange probability for nucleon collisions in air. Assuming a constant rate of energy loss b GeV/(g/cm²) for the fraction of the path in which the nucleon is charged, one obtains the mean energy loss $\varDelta E$ for a proton arriving at x:

$$\left.\begin{aligned} \Delta E &= \frac{2\,b}{1+\delta_0\, e^{-\frac{2\,xw}{\lambda}}} \int_0^x dy\, p\,(x,\,y) \\ &= \frac{b}{2}\left\{ x + \frac{1+\delta_0}{2}\,\frac{\lambda}{w}\,\frac{1-e^{-\frac{2\,xw}{\lambda}}}{1+\delta_0\, e^{-\frac{2\,xw}{\lambda}}} \right\} \\ \Delta E &\approx \frac{b}{1}\left[x + \frac{1+\delta_0}{2}\,\frac{\lambda}{w} \right] \quad \text{for} \quad 2\,x \gg \frac{\lambda}{w}. \end{aligned}\right\} \tag{D.2}$$

Using $b = 2\ \mathrm{MeV/(g/cm^2)}$ and the constants given in the text ($\lambda = 75\ \mathrm{g/cm^2}$, $\delta_0 = 0.74$, $w = 0.30$), one finds that the average proton has lost

1.25 GeV when reaching sea level and
0.95 GeV when reaching mountain altitude (700 $\mathrm{g/cm^2}$).

The corrected flux of protons in the atmosphere is therefore given by

$$N_p\,(x,\,E) = \frac{S_0\, e^{-x/\Lambda}}{2\,(E-M+\Delta E)^{\gamma+1}} \left[1+\delta_0\, e^{-\frac{2\,xw}{\lambda}\left(1-\frac{\lambda}{\Lambda}\right)} \right]. \tag{D.3}$$

This relation is plotted in Fig. 2.

b) Contributions from recoil nucleons.

In order to compare the flux of low energy protons near sea level with the calculated flux, one must add to the flux of extraterrestrial protons given by eq. (D.3) a contribution from recoils, i. e. nucleons of terrestrial origin which originally formed part of air molecules. These recoils receive kinetic energies upto ~ 3 GeV when the collision energy is low ($\lesssim 10$ GeV); in more energetic collisions, the energy which a recoiling baryon receives approaches a small constant value, and the resulting recoil nucleon has a maximum energy

$$E_{\max} = \left[\frac{M_B}{2\,\eta'\,M_P}\,(1+P_\perp^2) + \frac{\eta'\,M_P}{2\,M_B} \right] \left[\frac{M_B^2 + M_P^2 - n_B^2\, m_\pi^2}{2\,M_B} \right] (1+\beta_B\,\beta_P). \tag{D.4}$$

Here, $P_\perp$ is the transverse momentum taken up by the baryon in the excitation process, and $1-\eta'$ is the fractional collision energy used up in the creation of particles via the pionization process. The first bracket represents the energy in rest mass units of the baryon in the L-system, and the second bracket represents the energy of the nucleon in the baryon rest system, on the assumption that it decays into n_B pions of equal energy. β_B and β_P are the corresponding velocities of the baryon and the nucleon.

Setting $P_\perp \approx 500$ MeV/c and using the values derived in the text ($\eta' = 0.75$, $M_B = 2200$ MeV and $n_B = 3.5$), one finds a recoil kinetic energy

$$T_{\max} \approx 3.0 \text{ GeV};$$

if no excitation takes place ($M_B = M_P$, $n = 0$, $\beta_P = 0$)

$$T_{\max} \approx 250 \text{ MeV}.$$

Thus it is to be expected that, when reaching sea level, recoil nucleons will make a contribution to the proton flux mainly in the non-relativistic region. Fig. 2 shows that, in the energy region 0–500 MeV, there is in fact an excess of observed protons over and above the flux of extra-terrestrial protons calculated according to eq. (D.3). The excess is of the correct order of magnitude to be attributed to particles of terrestrial origin.

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